Using math misconceptions in teaching

Cornell Math Teaching Seminar 4/21/2021

Benjamin Thompson

bgt37@cornell.edu

he/him

Conceptual mistakes

A question in the MATH1920 Prelim 1 a few months ago:

3. Let $\mathbf{r_1}(t) = \langle t, 2t, t^2 \rangle$ and $\mathbf{r_2}(t) = \langle t^2, 1-t, 2-t^2 \rangle$ be paths.

   (a) Recall that two paths intersect if there is a point P that lies on both curves. Does $\mathbf{r_1}$ intersect $\mathbf{r_2}$? If so, find the point(s) of intersection.

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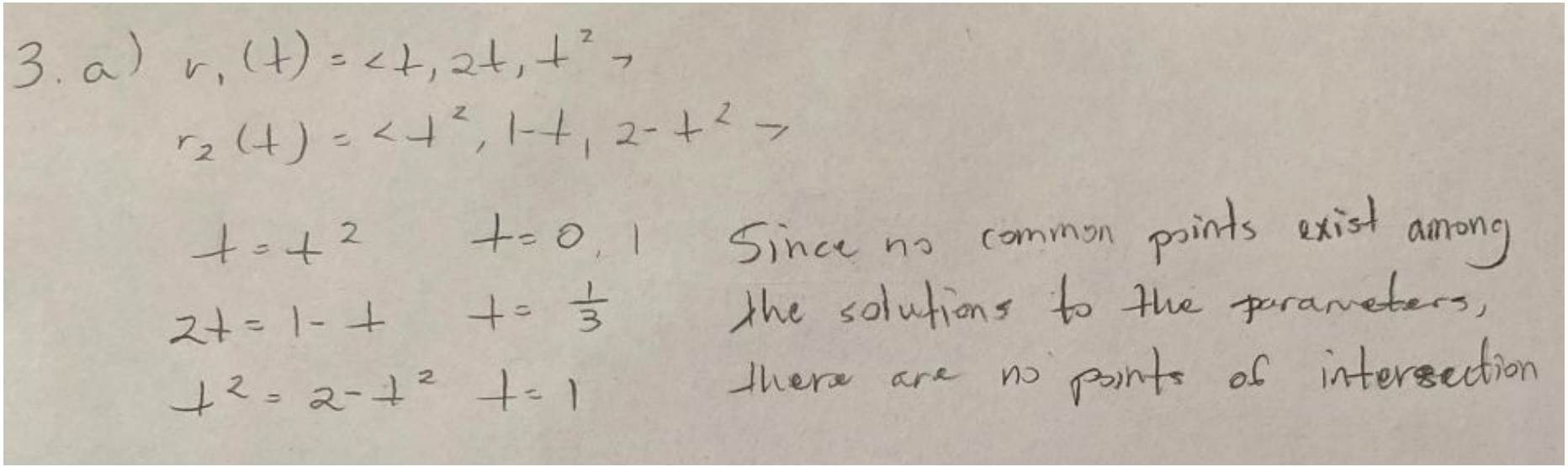

Question

Is it worthwhile to spend time covering common misconceptions?

Outline

- I’ll discuss and summarize some results from a PhD thesis about misconceptions and multimedia education
- Jamboard activity: identify some common misconceptions in math courses
- Jamboard activity: create a sketch of a dialogue

PhD Thesis Author Bio

- Derek Muller best-known for his popular YouTube channel “Veritasium” (8.8 Million subscribers as of April 2021)
- Very popular videos; our Steven Strogatz appeared in his video on synchronization last month!
- Wrote PhD Thesis on multimedia in physics education at University of Sydney in 2008.
- Worked on Australia’s ABC science journalism show *Catalyst* in early 2010s

From “How to Understand the Black Hole Image” on Youtube

From “The Secret of Synchronization” on Youtube

PhD Thesis Overview

- Title: “Designing Effective Multimedia for Physics Education”
- First line of abstract: “This thesis summarizes a series of investigations into how multimedia can be designed to promote the learning of physics.”
- Chapters 8 - 10 involve creating, and then evaluating, multimedia for quantum and Newtonian mechanics
- I’ll summarize the studies in these chapters

Muller’s thesis is available on his website:
https://www.veritasium.com/about

Chapter 8 Summary

- Goal: “to incorporate dialogue into a multimedia resource and assess its effectiveness for changing student conceptions.” (p141)
- Hypothesis: “this might be an effective way to confront alternative conceptions.” (p141)
- Experiment summary:
- Students did a pre-test which evaluated students’ understanding of quantum tunneling
- Students were randomly assigned one of two videos
- Students then took a post-test which was the same as the pre-test
- The test results were compared

Chapter 8: What was in the videos?

- Dialogue: “a video simulating the discussion that might take place between a student and a TA”
- In the video the TA questioned parts of the students reasoning so that the student “identified and resolved inconsistencies”
- Exposition: “summarized the correct physics information in the dialogue but without alternative conceptions” (p147)

Chapter 8: Results

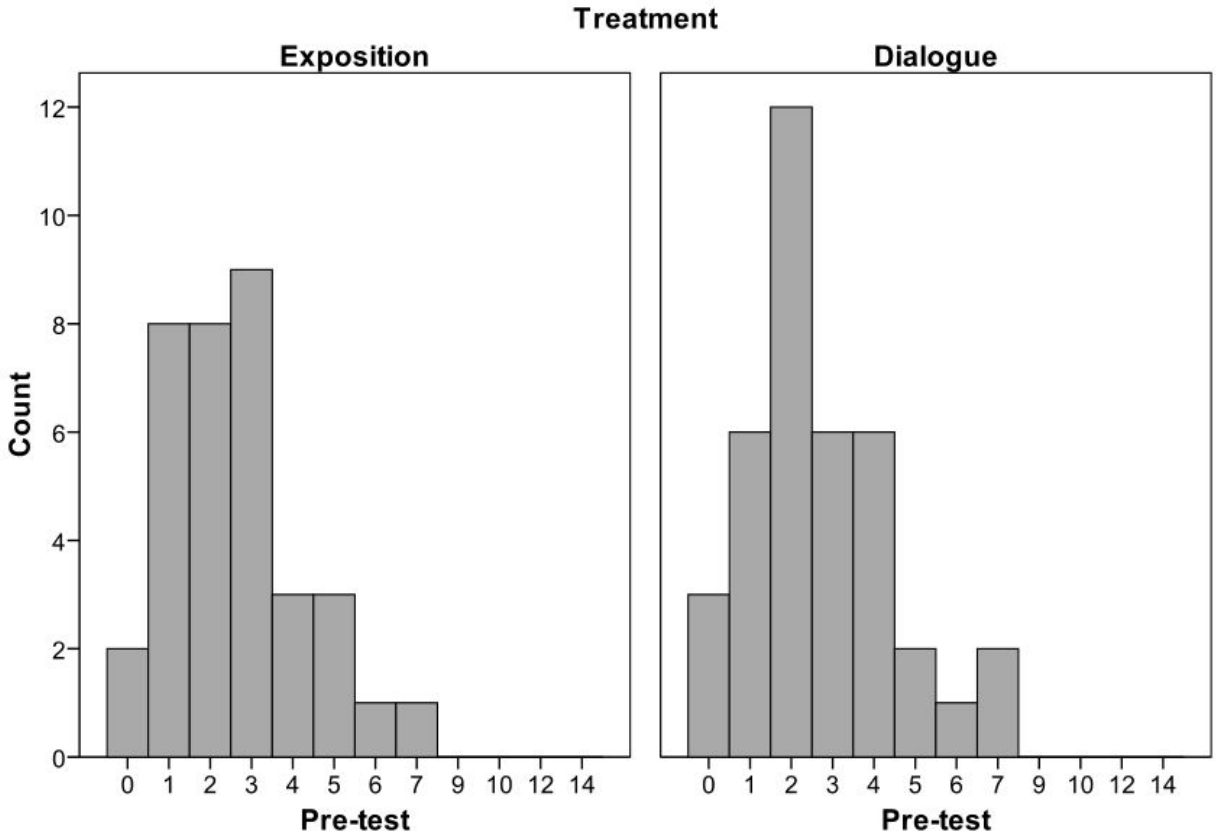

Figure 8.1, p150

Chapter 8: Results

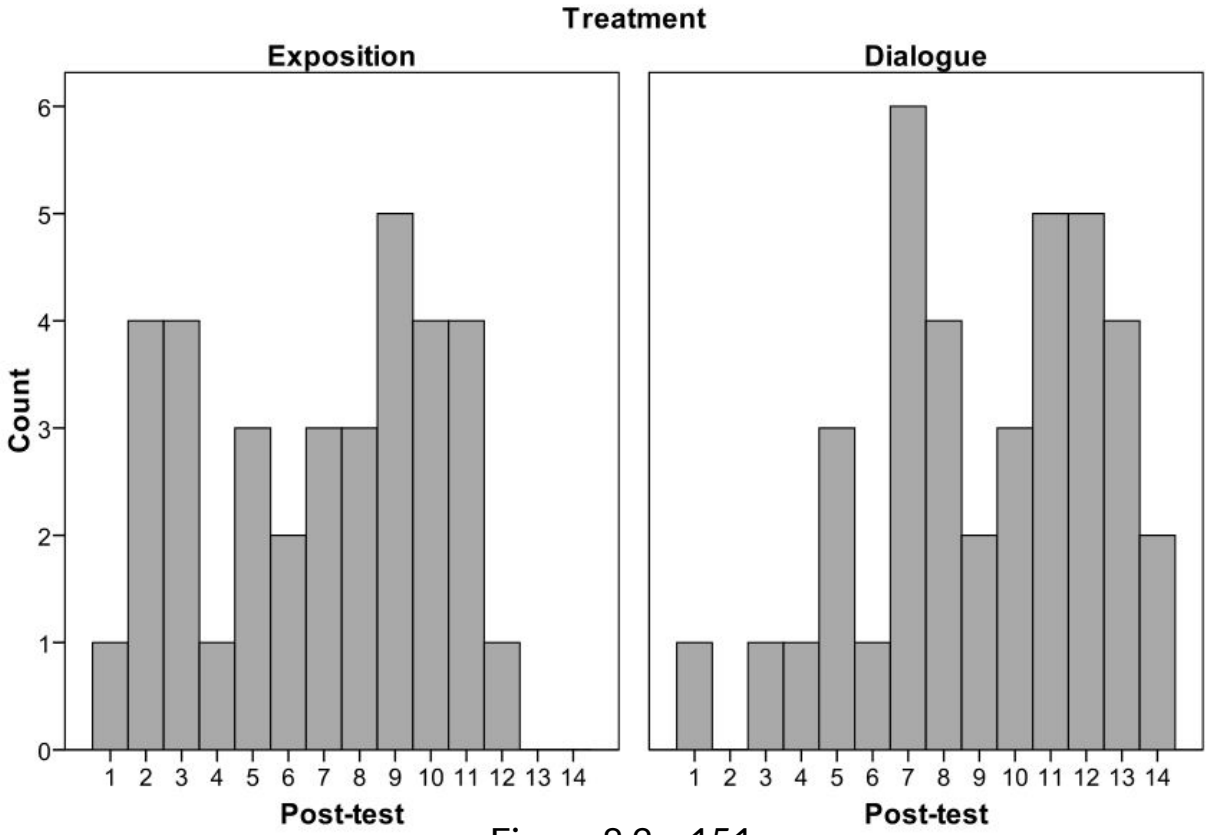

Figure 8.2, p151

Chapter 8: Conclusion

- In quantum mechanics teaching:
- “Students learned better with multimedia when common alternative conceptions were presented in a dialogue format than when only correct information was presented in a lecture style.” (p161)

Additional Question / Concern:

- “Is dialogue in an essential feature or could misconceptions be stated and refuted by a single speaker with equal effectiveness?” (p163)
- The dialogue was longer than the exposition: was the test-score difference simply due to a longer time spent with the content?
- Two new types of videos were created to address these concerns

Addressing the Additional Question / Concern:

- New video types:
- Refutation: “alternative conceptions were raised in a lecture style”
- Extended Exposition: “additional interesting and related material was added to the Exposition” to make it as long as the Dialogue
- The effectiveness of these is examined in Chapter 9

Chapter 9 Overview

- Looks at misconceptions in Newtonian mechanics (these have been studied more)
- Studies how the new video types improve understanding
- Students were divided into physics streams (fundamentals, regular, advanced)
- Tests were the same set of 26 multiple-choice questions from a standard set used in physics education research (Force and Motion Conceptual Evaluation (FMCE))

Chapter 9 Overview

- Experiment similar to Chapter 8: pre-test, random video, post-test

Video type	Exposition	Dialogue	Refutation	Extended Exposition
Number of speakers	1	2	1	1
Length	7:02	11:22	9:33	11:22
Address misconceptions	No	Yes	Yes	No

Video summary: Table 9.1, p172

Chapter 9: Results

Which videos correspond to which test gain in test results?

Options:

- Exposition
- Dialogue
- Refutation
- Extended Exposition

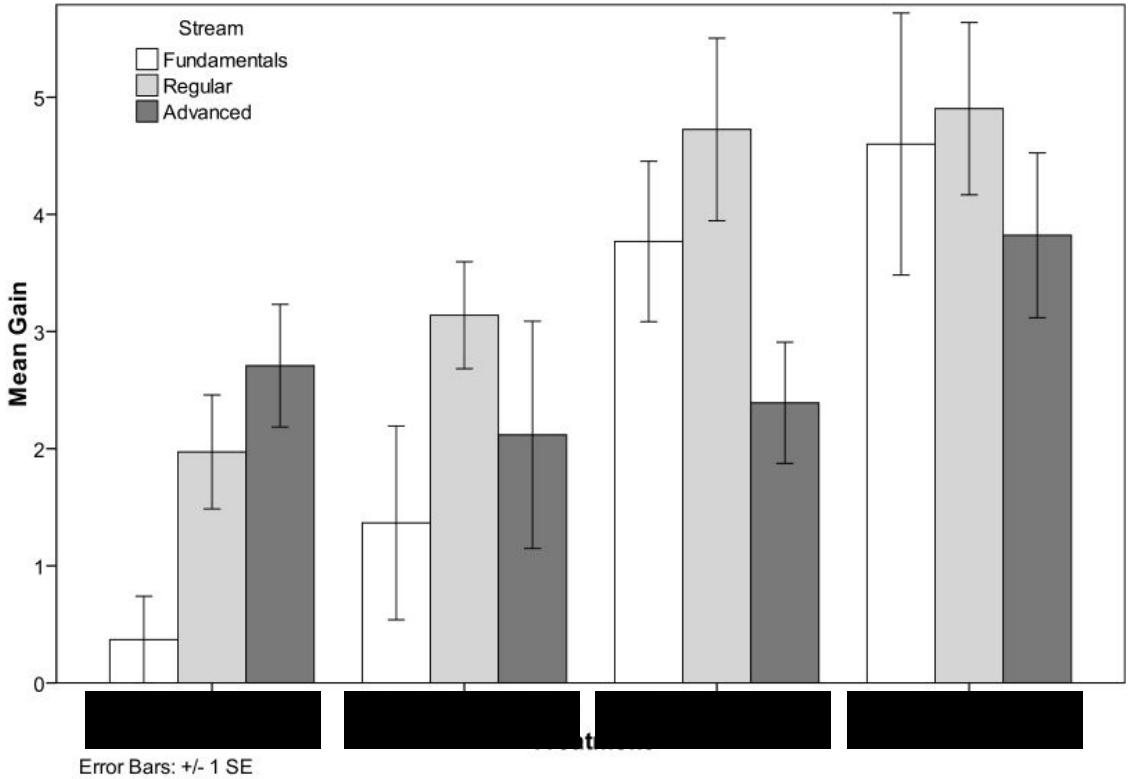

Figure 9.1, p175

Chapter 9: Results

Which do you think gave the largest gain in pre/post-test scores? [Type in chat]

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- Exposition
- Dialogue
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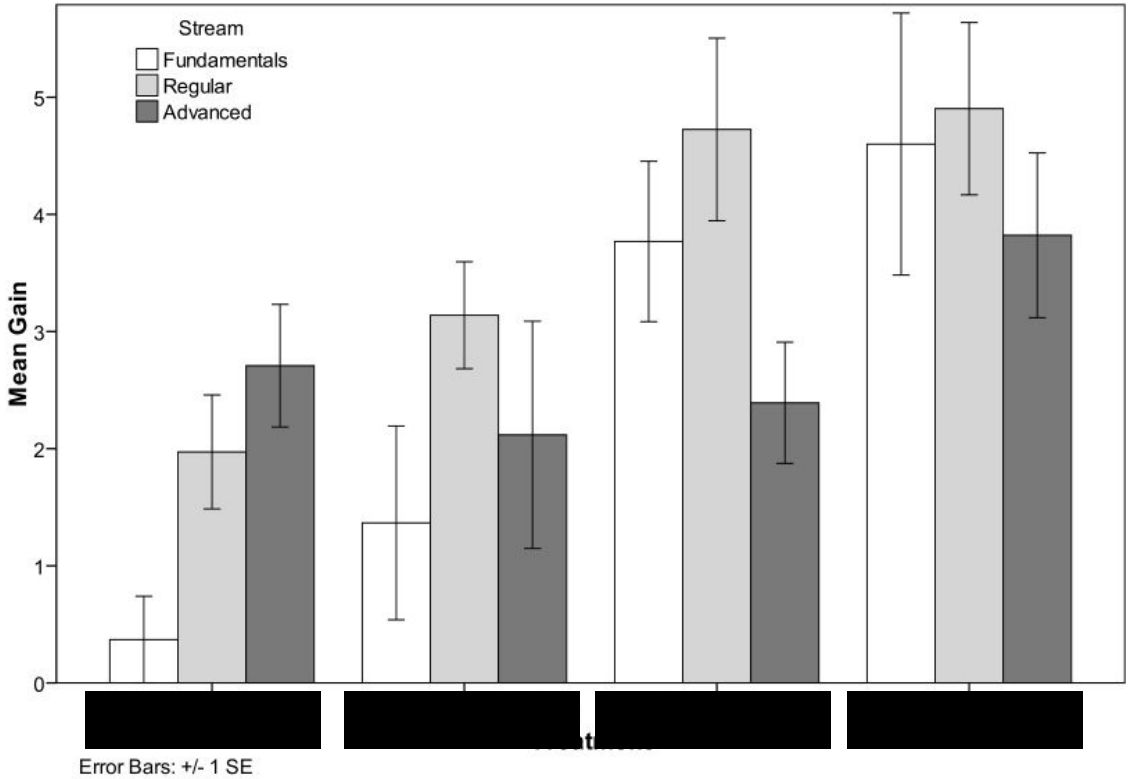

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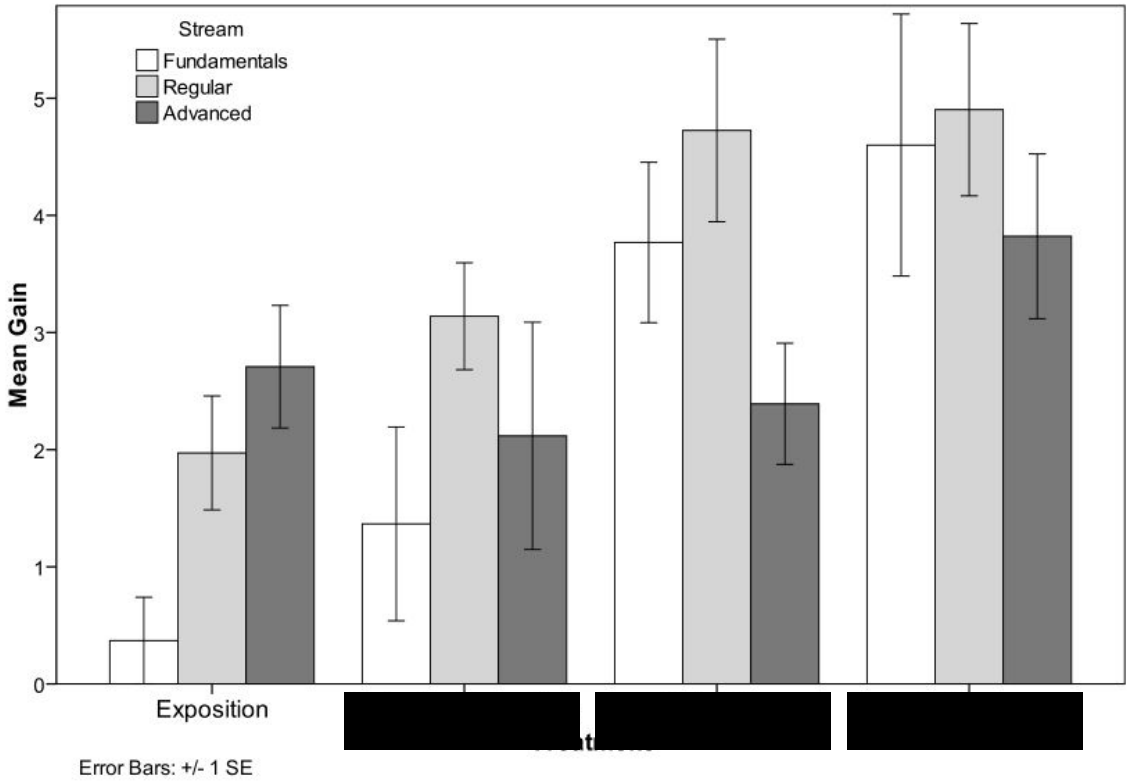

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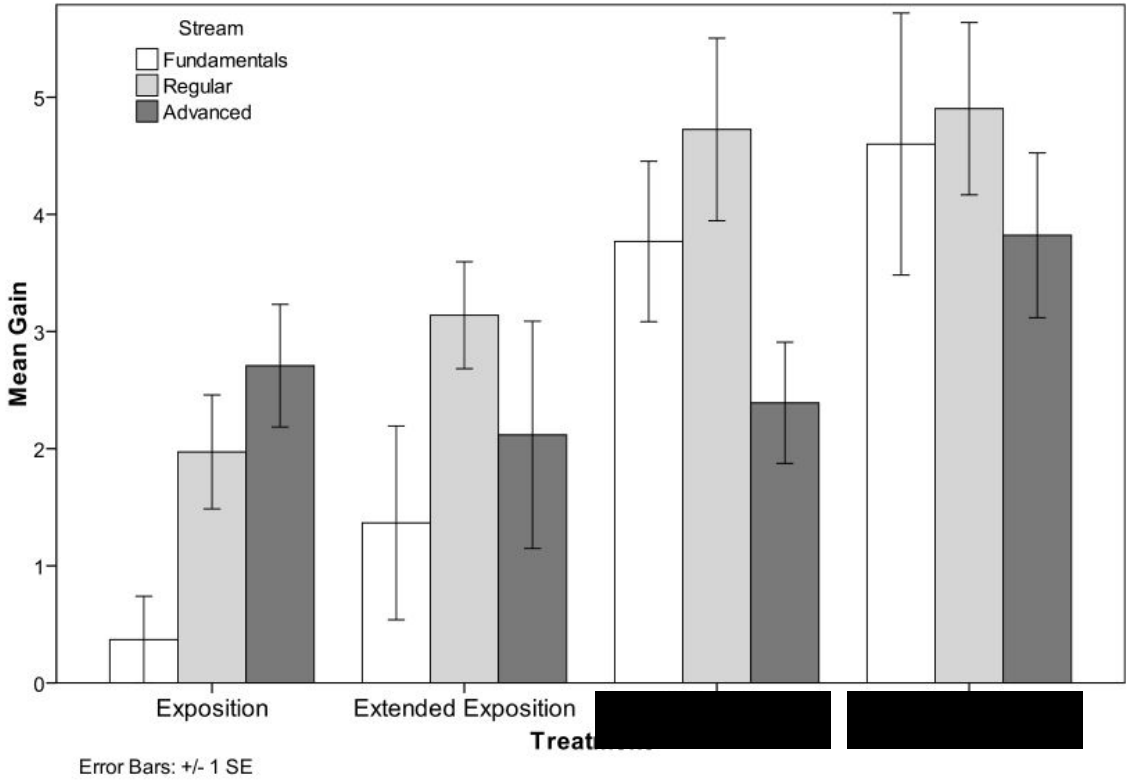

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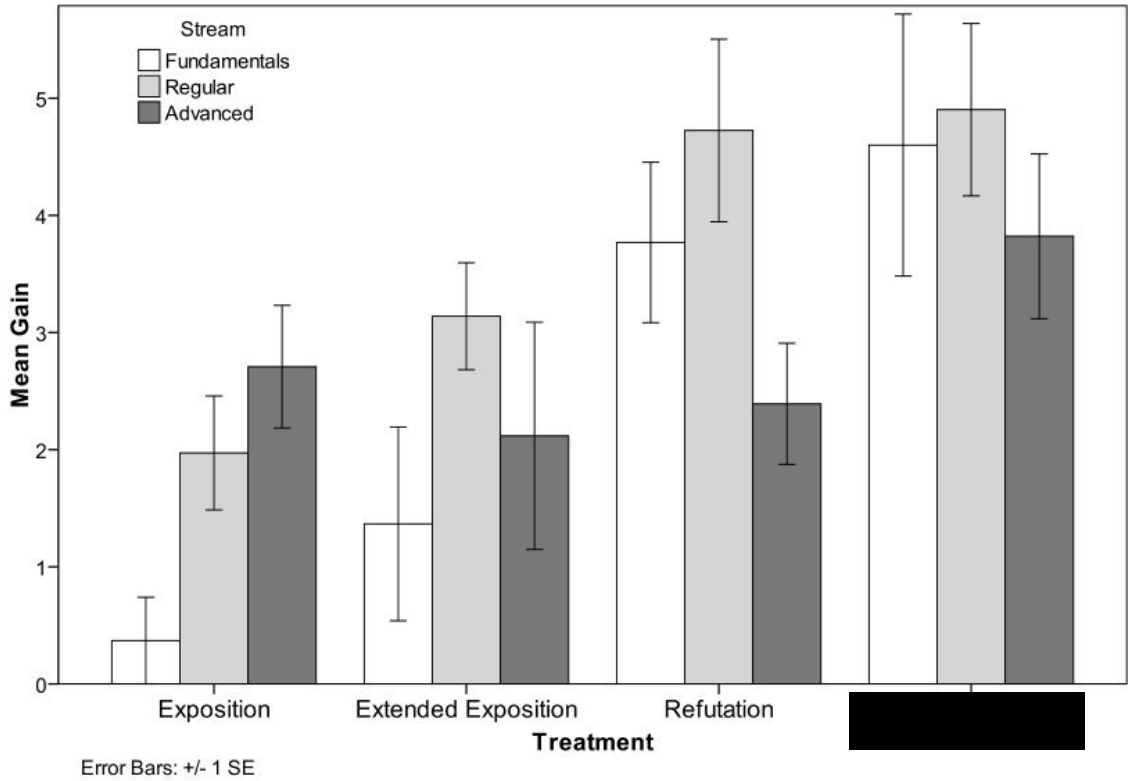

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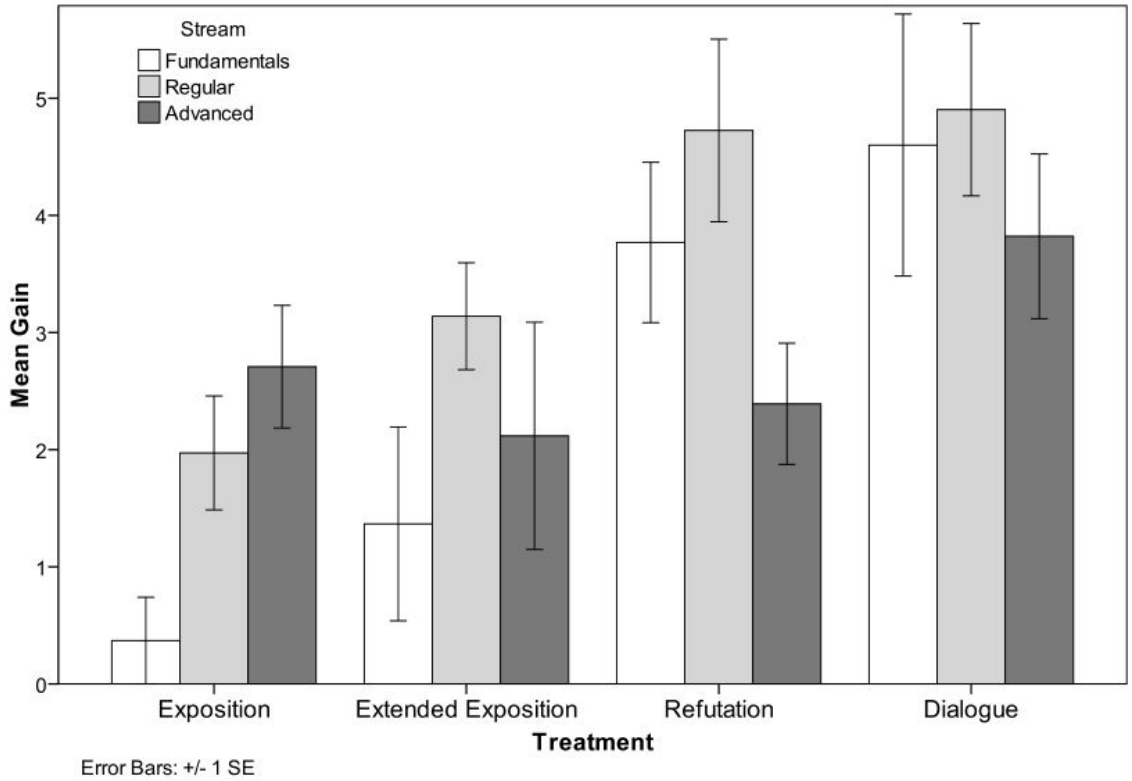


Figure 9.1, p175

Did students feel more confident with their answers?

- Students were asked to rate their confidence with the material after each test
- No significant difference!

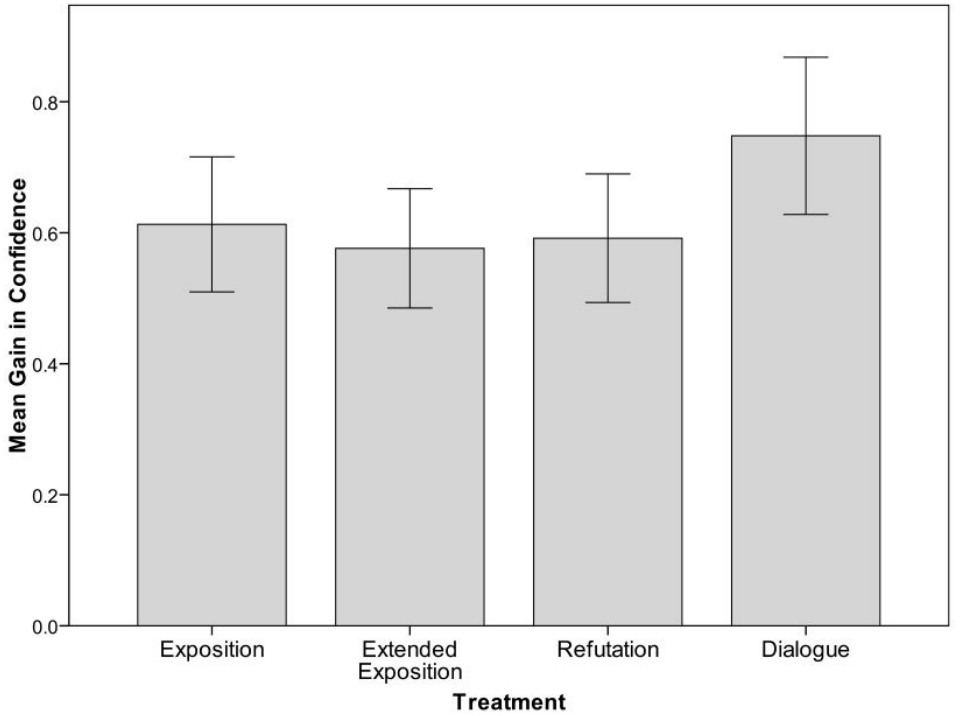


Figure 9.2, p176

Chapter 9: Conclusion

- “Results show that overall students achieved greater gains by watching a treatment that addressed misconceptions than one which presented only scientific information.” (p177)
- “The explicit discussion of misconceptions seems to be an effective instructional strategy whether students actually hold the misconceptions or not.” (p178)

Chapter 10: Summary

- Asks four questions based on results from chapter 9, mainly about mental effort:

1. “How does mental effort spent / post-test scores compare between:
   a. Dialogue and Exposition?
   b. misconception-based and non-misconception based videos?
2. How does the presence of a pre-test effect mental effort spent / post-test scores ?
3. For advanced students, is there any difference in effectiveness between Dialogue and Refutation?” (p182-183)

Chapter 10: Question 1

- “A t-test revealed that Dialogue students reported investing significantly more mental effort than Exposition students while watching the multimedia.” (p186)

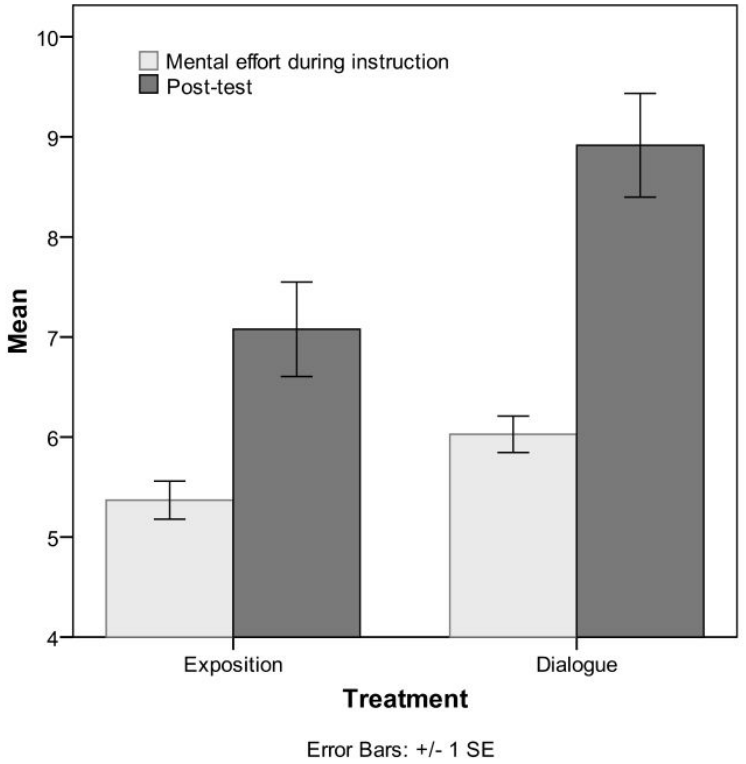


Figure 10.1, p187

Chapter 10: Question 2

- The extended exposition was replaced with a video with worked examples

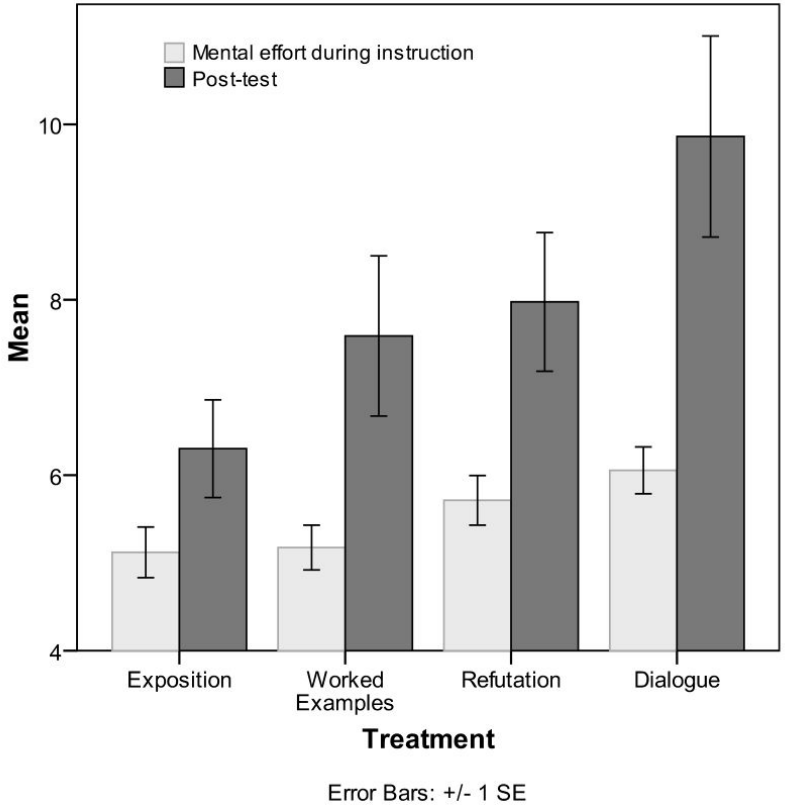


Figure 10.3, p190

Chapter 10: Question 2

- “T-tests revealed that students who watched [videos with] higher alternative conceptions invested significantly higher mental effort and achieved significantly higher post-test scores.” (p188)

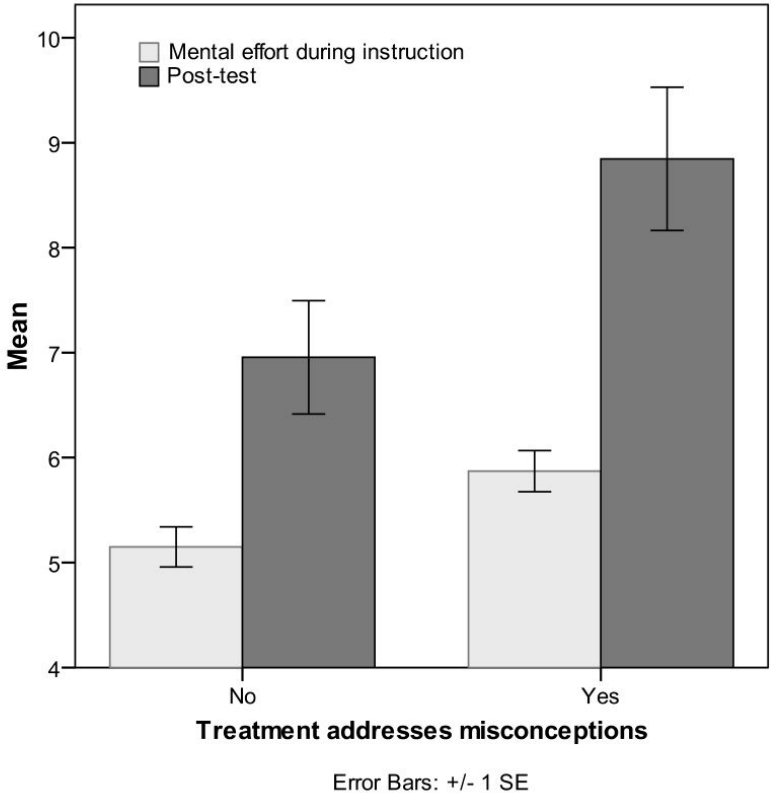


Figure 10.2, p189

Chapter 10: Question 3

- "Students who completed the pre-test took less time to complete the post-test but more time to watch the [videos]" (p191)

Pre-test condition	Treatment	Sample size (n)	Time spent (min) on multimedia		Time spent (min) on post-test	
			M	SD	M	SD
Pre-test	Exposition	42	14.1	11.3	10.2	6.7
	Dialogue	57	15.2	7.5	11.9	7.4
No pre-test	Exposition	57	10.5	7.5	19.6	7.2
	Dialogue	57	12.9	2.8	19.6	9.1

Table 10.2, p191

Chapter 10: Question 4

- "Mental effort and gain scores were nearly identical for the two [advanced] groups" (p193)

Treatment	Pre-test		Post-test		Gain		Mental effort invested during: instruction		Mental effort invested during: the post-test	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Dialogue	17.5	7.1	20.3	6.5	2.84	4.22	4.59	1.74	4.05	1.41
Refutation	18.9	6.3	21.8	4.7	2.86	3.55	4.66	1.85	4.02	1.77

Table 10.4, p193

Chapter 10: Conclusion

- “In comparing the two misconception-based multimedia treatments, the Refutation and Dialogue seem to be equally effective at promoting conceptual change.” (p203)
- “However, interviews suggest that in addition to remedying alternative conceptions, the Dialogue treatment may provide additional benefits.” (p203)

Jamboard Exercise (10min):

- Identify some common misconceptions in your subject
- What are some ways these could be incorporated into:
  - Lectures?
  - Recitations?
  - Assessment?
- How can we identify common misconceptions / record them for future use in the courses?

Some things I've tried this semester

- I came up with incorrect solutions to worksheet problems, and asked students to identify the errors / fix them
- More often than not, when I asked students to point the errors out, they responded
- I get the feeling some students zone out, and only pay attention when we're fixing them
- I find it difficult to predict conceptual errors

Example of corrected solutions in one of my MATH1920 worksheets

2. Assume that $||\mathbf{v}|| = 3$, $||\mathbf{w}|| = 5$, and the angle between $\mathbf{v}$ and $\mathbf{w}$ is $\theta = \frac{\pi}{3}$.

   (a) Make a sketch of two vectors $\mathbf{v}$ and $\mathbf{w}$ which satisfy the information given above.

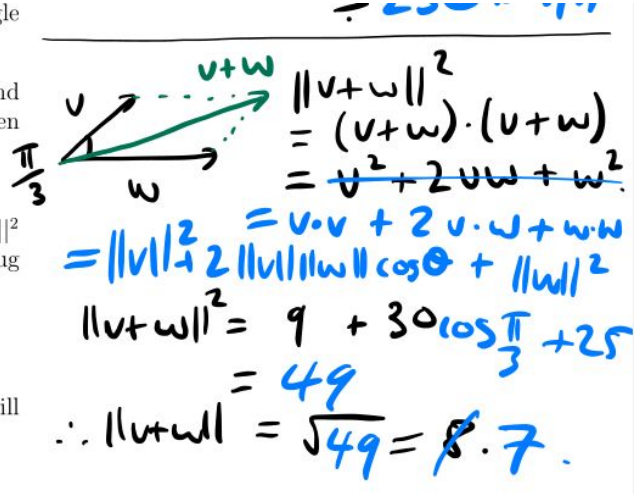
   (b) Sketch the vector $\mathbf{v} + \mathbf{w}$.

   (c) Recall that $\mathbf{v} \cdot \mathbf{v} = ||\mathbf{v}||^2$. Find $||\mathbf{v} + \mathbf{w}||^2$ in terms of $||\mathbf{v}||$, $||\mathbf{w}||$, and θ (don't plug in numbers yet!).

   (d) Calculate $||\mathbf{v} + \mathbf{w}||$.

   (e) Does $||\mathbf{v} + \mathbf{w}|| = ||\mathbf{v}|| + ||\mathbf{w}||$?

   (f) *Challenge:* For what vectors $\mathbf{v}$ and $\mathbf{w}$ will $||\mathbf{v} + \mathbf{w}|| = ||\mathbf{v}|| + ||\mathbf{w}||$?



Example Dialogue (from a MATH1110 worksheet)

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   $$f'(x) = \frac{x\cos(x) - \sin(x)}{x^2}.$$

   Evaluating at $x = 0$ *then produces* $f'(0) = 0$.

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SOCRATES: *But would it not be easier to apply l'Hôpital's Rule? Since the limits of the numerator and denominator are both zero at $x = 0$, l'Hôpital's Rule implies*

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So evaluating $f'(x)$ at $x = 0$ would actually produce $f'(0) = 1$.

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   So evaluating $f'(x)$ at $x = 0$ would actually produce $f'(0) = 1$.

   Explain any mistakes Plato and Socrates have made. Does this discussion contradict l'Hôpital's Rule?

Jamboard Exercise (10min):

- Choose one of your previously identified misconceptions, and write a sketch of a dialogue to communicate the misconception
- Would a dialogue be the most effective way to communicate the idea? If not, what else could be used?

Further work

- The question bank that Muller used --- does something similar exist in math? (I haven't yet explored much of the literature.)
- Would it be worthwhile creating such a resource for the standard courses?