Counting coned squares in Preferential Attachment Graphs

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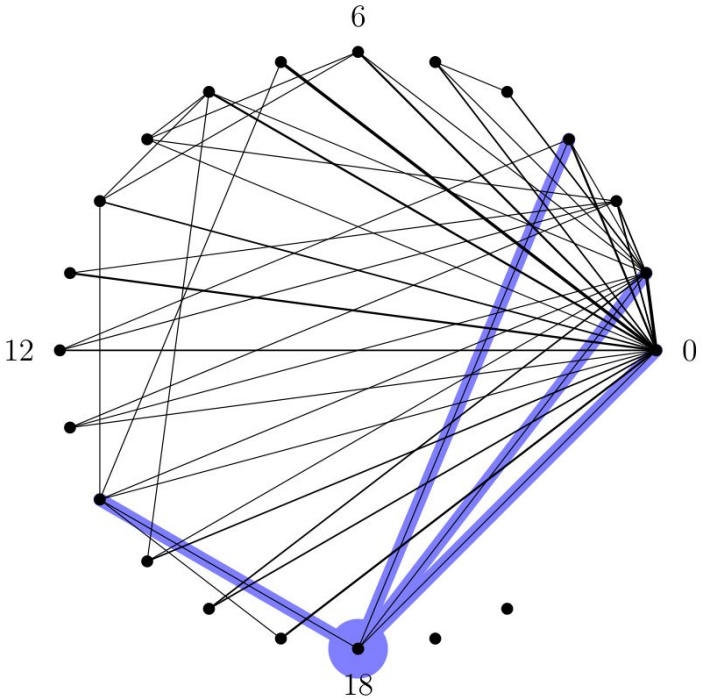
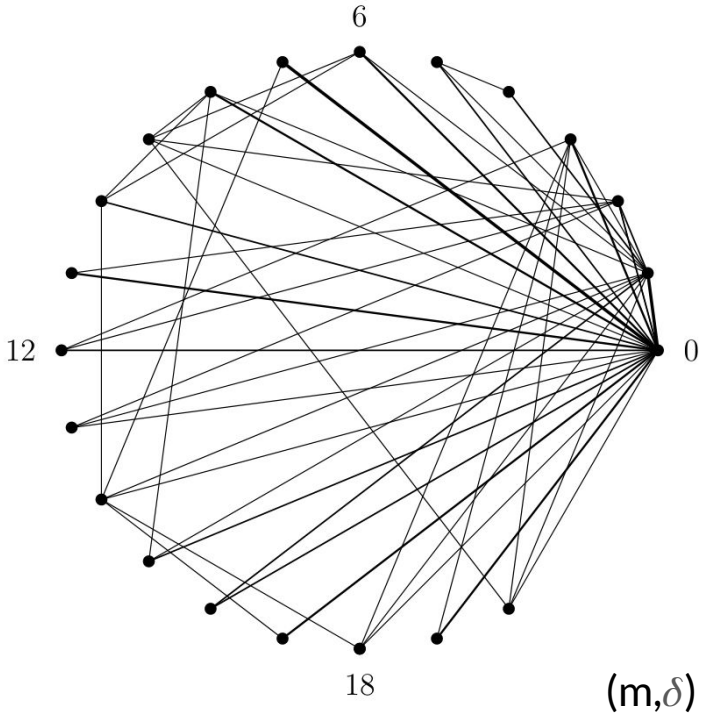
Constructing a Preferential Attachment Graph (PAG)

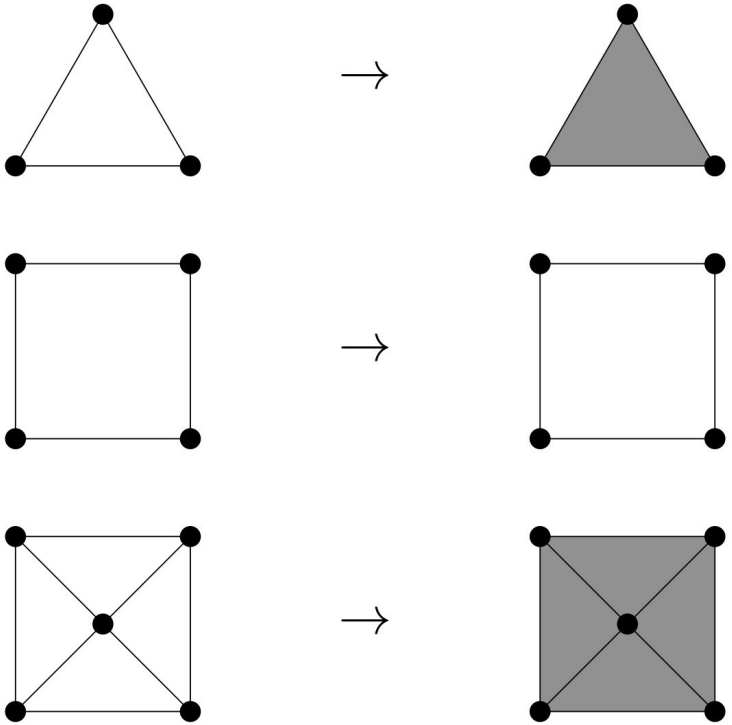




(m,δ) = (4,-3)

- Edges are attached to more popular nodes.

Coned squares in the clique complex of a PAG



Counting these (almost) give an upper bound.

Relationship of coned squares and Betti numbers

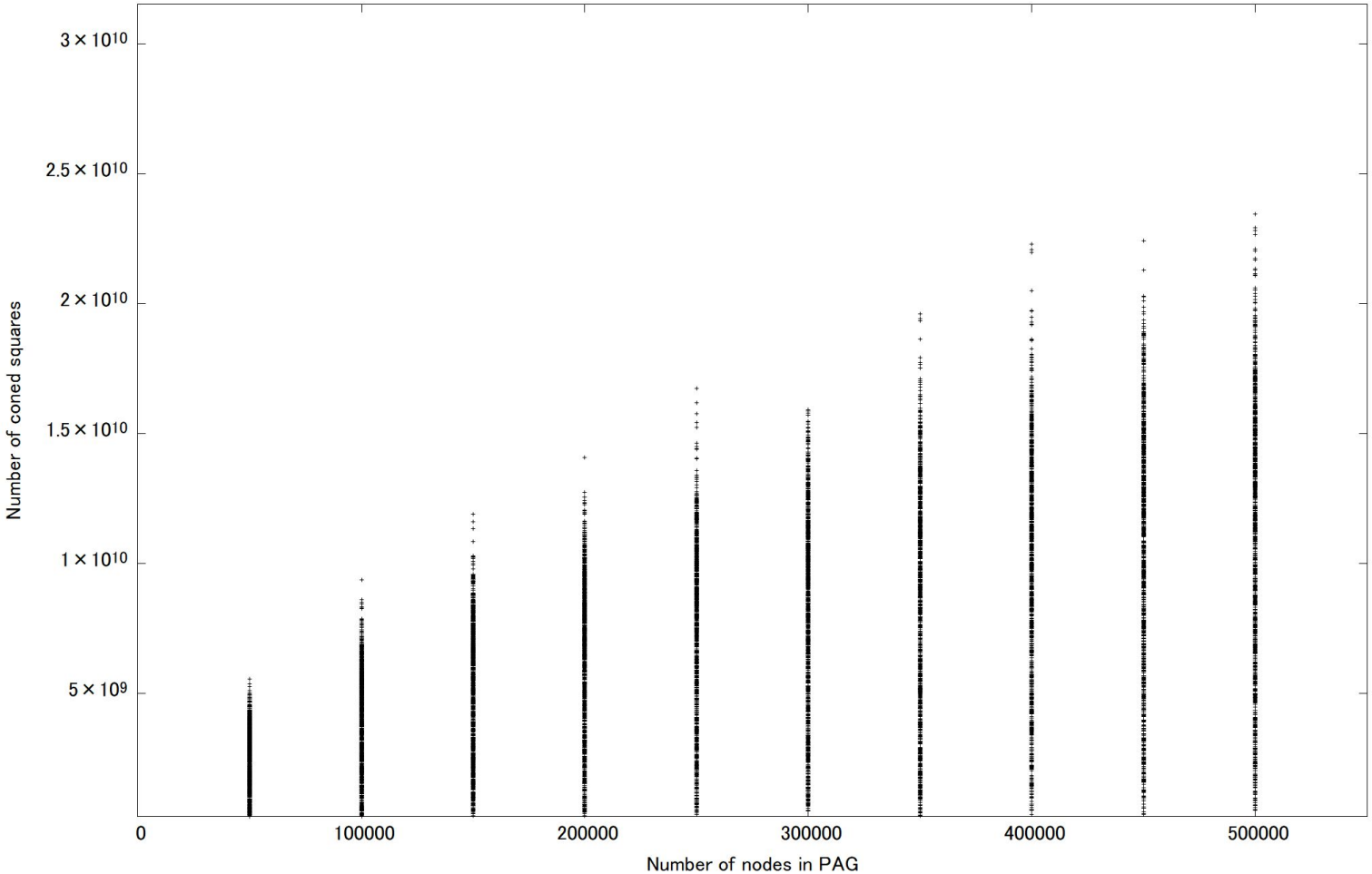
- Garavaglia and Steghuis (2019) proved the growth rate of expected number of coned squares:
- For q > 1, χ(m, δ), and some constants c, C:

$$c_1 T^{1-4\chi} \leq E[\#\text{coned squares}] \leq C_1 T^{1-4\chi}$$

- Siu C. et al. (2023+) proved the growth rate of the expected Betti number is similar:
- For q >1 and same χ:

$$c_2 T^{1-4\chi} \leq E[\beta_2] \leq C_2 T^{1-4\chi}$$

Coned squares vs node size, (m,d) = (15,-13), 1000 trials



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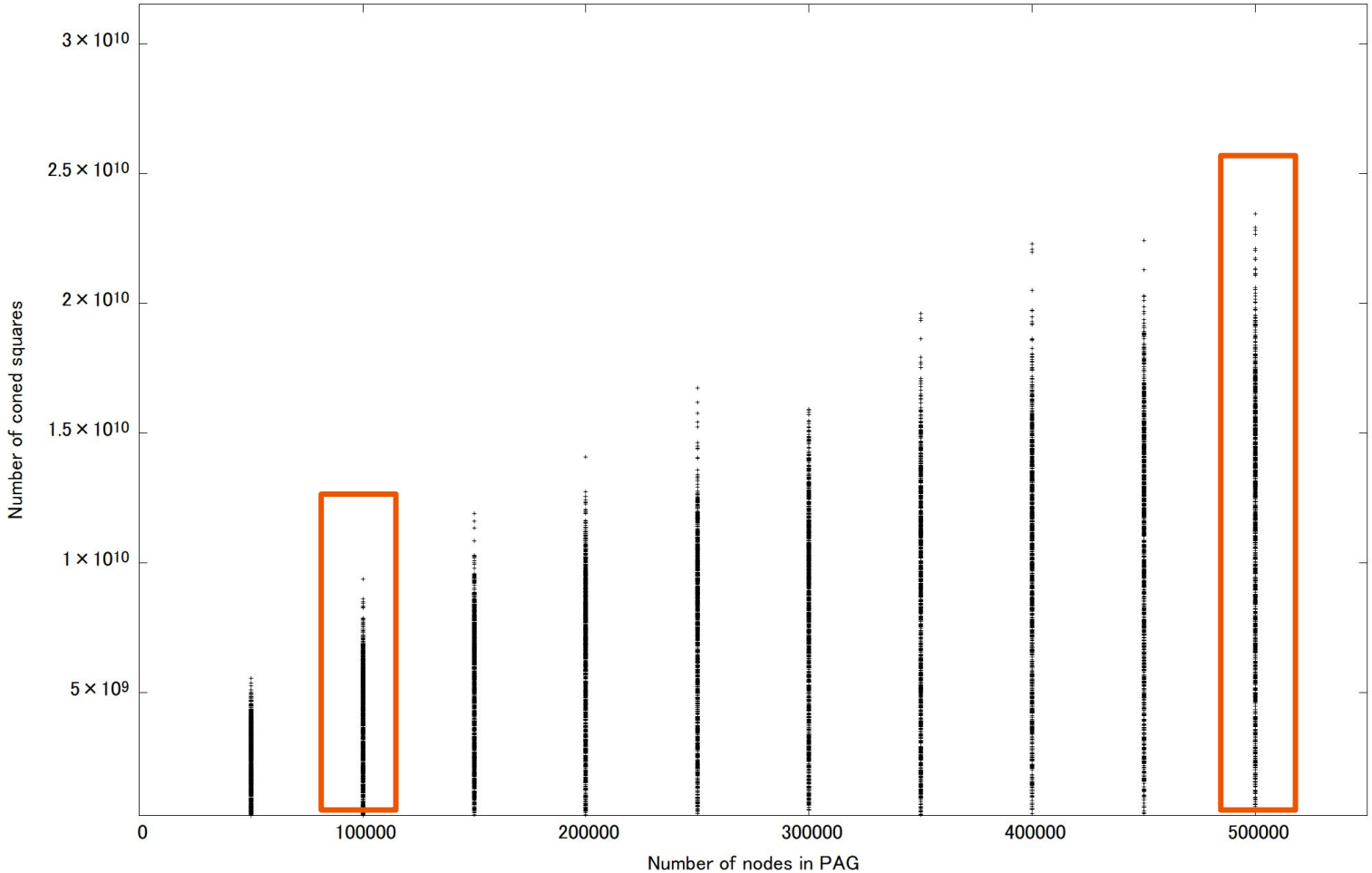
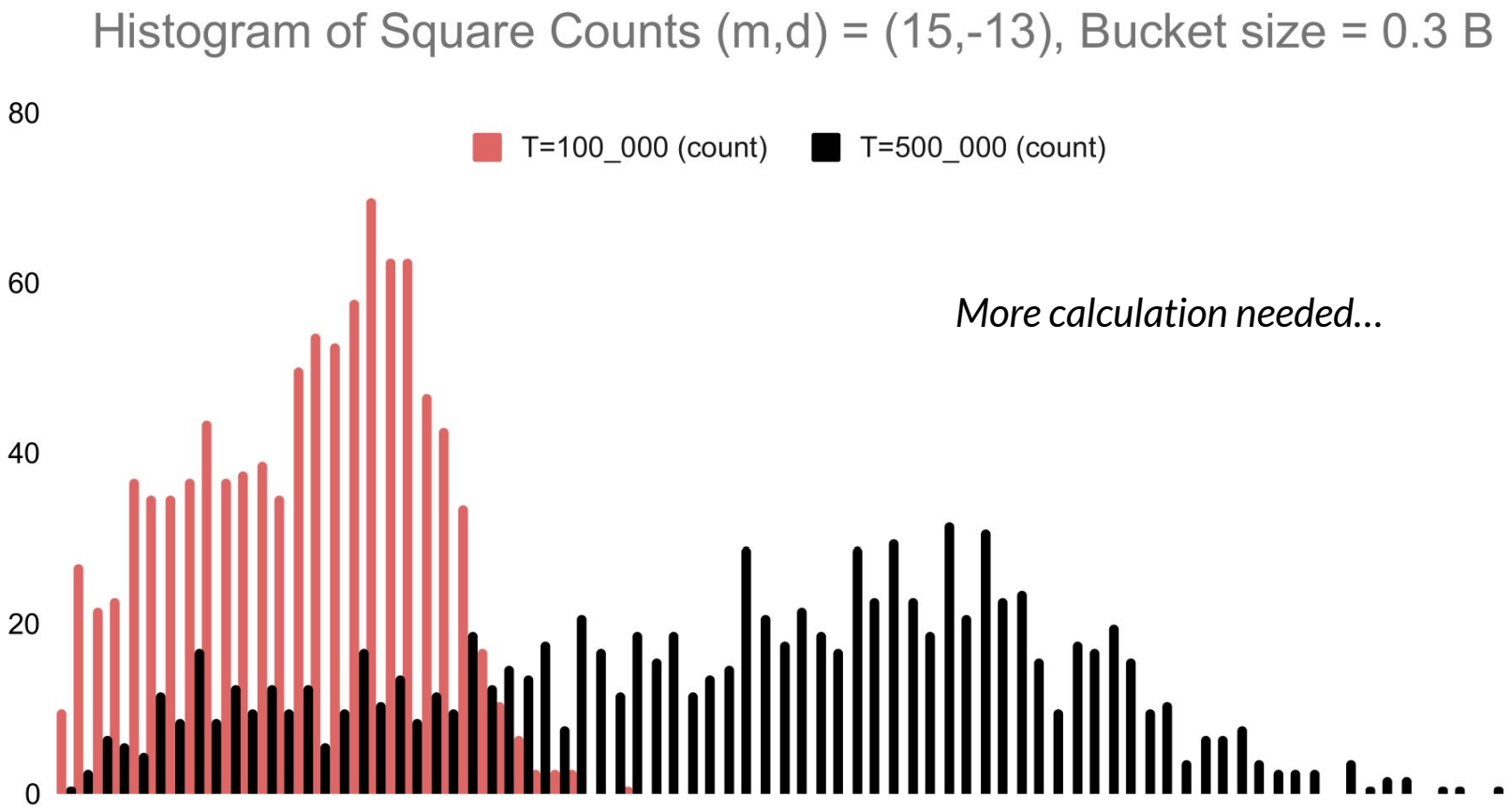


Histograms of PAGs with #nodes = 100_000, 500_000



Some new(ish) high-performance C/C++ alternatives

- Used Odin instead of C++ for the calculation
- Odin is a systems-level programming language
- Strongly typed / high quality compile errors
- Great allocators→ no need for Valgrind
- Calculation was simple in Odin (<400 lines)
- Fast (<24h, 10,000 graphs with > 1M edges)
- I enjoy using Odin and Zig, you may like them too
- Odin examples: github.com/bg-thompson

My experience with C++

(I love this language.)

(I think this is great too.)