MATH1106DIS 208 Week 9 (#2) 4/10/2020

Discussion Outline

- Worksheets

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Office hours: 4:15–6:15 Thu

Upcoming Assessment

- Homework 8 (Monday, 4/13)
- Reading Quiz 18 (Monday, 4/13)

Last time: - Integrating functions

This time: - Finding & describing equilibria

— Solutions to the worksheet today will viewable on Youtube tonight

This is a good time to get familiar with logs again.

Questions: What is e^x?

(e.g, what is e^1 to 1 decimal place? WITHOUT using a calculator)

What is $\ln x$?

Why is $\ln(x) + \ln(y) = \ln(xy)$?

What is e^x ?

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

?

$$= \lim_{N \to \infty} \sum_{n=0}^{N} \frac{x^n}{n!}$$

$$\sum_{k=0}^{3} 2k = 2k + 2k + 2k + 2k$$

$$k=0 \qquad k=1 \qquad k=2 \qquad k=3$$

$$= 2\cdot 0 + 2\cdot 1 + 2\cdot 2 + 2\cdot 3$$

$$= 2(0+1+2+3)$$

$$= 2(6) = 12.$$

$$\prod_{k=0}^{10} (k+5) = (0+5)\cdot(1+5)\cdot(2+5)\cdots(10+5)$$

... digression.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$n! := n\cdot(n-1)\cdot(n-2)\cdots$$

$$2! = 2\cdot 1 = 2$$

$$4! = 4\cdot 3\cdot 2 = 4\cdot 6 = 24$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^1 = \frac{1^0}{0!} + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} + \cdots$$

$$= \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \cdots$$

$$\simeq 2 + 1/2 + 1/6 + 1/24 + \cancel{1/120} \quad \text{too small}$$

$$\approx 2.5 + 0.165 + 1/24$$

$$= 2.665 + 1/24 \approx 2.7$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{x+y} = e^x \cdot e^y \quad .$$

$$\sum_{n=0}^{\infty} \frac{(x+y)^n}{n!} = \left(\sum_{n=0}^{\infty} \frac{x^n}{n!}\right)\left(\sum_{n=0}^{\infty} \frac{y^n}{n!}\right)$$

Provable!

What is $\ln$? $\ln(ab) = \ln(a) + \ln(b)$

If $a = e^x$,

$\ln a = x$

$e^0 = 1 \rightarrow \ln(1) = 0$

$e^1 = e \rightarrow \ln e = 1$

Why does $\ln(ab) = \ln a + \ln b$?

Let $a, b > 0$.

Possible to write $a = e^{\tilde{a}}$ $\ln a = \tilde{a}$

$b = e^{\tilde{b}}$ $\ln b = \tilde{b}$

$$e^{\tilde{a}} \cdot e^{\tilde{b}} = e^{\tilde{a}+\tilde{b}}$$

$$\ln(ab) = \ln(e^{\tilde{a}} \cdot e^{\tilde{b}})$$

$$= \ln(e^{\tilde{a}+\tilde{b}})$$

$$= \tilde{a} + \tilde{b} = \ln a + \ln b.$$