

MATH 1106 DIS 208

Week '9' (#2)

4/10/2020

Discussion Outline

- Worksheets

Benjamin Thompson (he/him)

bgt37@cornell.edu

Office hours: 4:15 - 6:15 Thu

Upcoming Assessment

- Homework 8 (Monday, 4/13)
- Reading Quiz 18 (Monday, 4/13)

Last time: - Integrating functions

This time: - Finding & describing equilibria

- Solutions to the worksheet today will be viewable on YouTube tonight

This is a good time to get familiar with logs again.

Questions : What is e^x ?

(e.g, what is e^1 to 1 decimal place? **WITHOUT** using a calculator)

What is $\ln x$?

Why is $\ln(x) + \ln(y) = \ln(xy)$?

What is e^x ?

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$= \lim_{N \rightarrow \infty} \sum_{n=0}^N \frac{x^n}{n!}$$

↙ ?

$$\sum_{k=0}^3 2k = 2k \quad + \quad 2k \quad + \quad 2k \quad + \quad 2k$$

$k=0$ $k=1$ $k=2$ $k=3$

$$= 2 \cdot 0 \quad + \quad 2 \cdot 1 \quad + \quad 2 \cdot 2 \quad + \quad 2 \cdot 3$$

$$= 2(0 + 1 + 2 + 3)$$

$$= 2(6) = 12.$$

$$\prod_{k=0}^{10} (k+5) = (0+5) \cdot (1+5) \cdot (2+5) \cdot \dots \cdot (10+5)$$

... digression.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$n! := n \cdot (n-1) \cdot (n-2) \cdot \dots$$

$$2! = 2 \cdot 1 = 2$$

$$4! = 4 \cdot 3 \cdot 2 = 4 \cdot 6 = 24$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^1 = \frac{1^0}{0!} + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} + \dots$$

$$= \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \dots$$

$$\geq 2 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \dots$$

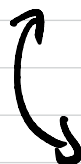
$$\approx 2.5 + 0.165 + \frac{1}{24}$$

too small

$$= 2.665 + \frac{1}{24} \approx 2.7$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{x+y} = e^x \cdot e^y$$


$$\sum_{n=0}^{\infty} \frac{(x+y)^n}{n!} = \left(\sum_{n=0}^{\infty} \frac{x^n}{n!} \right) \left(\sum_{n=0}^{\infty} \frac{y^n}{n!} \right)$$

Provable!

What is \ln ? $\ln(ab) = \ln(a) + \ln(b)$

$$\text{If } a = e^x,$$

$$\ln a = x$$

$$e^0 = 1 \rightarrow \ln(1) = 0$$

$$e^1 = e \rightarrow \ln e = 1$$

Why does $\ln(ab) = \ln a + \ln b$?

Let $a, b > 0$.

Possible to write $a = e^{\tilde{a}}$ $\ln a = \tilde{a}$

$b = e^{\tilde{b}}$ $\ln b = \tilde{b}$

$$e^{\tilde{a}} \cdot e^{\tilde{b}} = e^{\tilde{a} + \tilde{b}}$$

$$\ln(ab) = \ln(e^{\tilde{a}} \cdot e^{\tilde{b}}).$$

$$= \ln(e^{\tilde{a} + \tilde{b}})$$

$$= \tilde{a} + \tilde{b} = \ln a + \ln b.$$