

MATH 1100

Wk2 Discussions 9/8/20

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Before we start

- There's a good chance some of us will have to quarantine at some point ...

you can attend any discussion section online in this case.

Discussion Outline:

- Why study math?
- Mathematics Mythbusting
 - Published math is always correct
 - Being correct is everything
 - Mathematics is done by solitary geniuses
- Math support groups
- ↳ Plagiarism & Cheating
- Letting Cornell know about horrible people.

Why Study Math?

Why Study Math?

Some reasons: (not all!)

- It's used throughout science
- It's a CAS prereq. (valid reason)
- Problem solving is fun
- You can use it to prove things about the world that are impossible to prove without math
- Once something's proved it's true forever

Question

Prove that at any given time, there are two opposite points on the surface of the Earth with the same temperature and pressure.

(2 min)

Proof:

Consider the xy -plane. Let the x -axis denote temperature, and the y -axis denote pressure. Then at each point on the Earth, the temperature and pressure at that point is represented by a point in the xy plane. The Earth is roughly spherical, so we have a function

$$f: S^2 \rightarrow \mathbb{R}^2,$$

where S^2 is the sphere and \mathbb{R}^2 is the plane, defined by taking a point to its temperature & pressure.

Now since temperature and pressure vary continuously over the surface of the Earth,

$$f: S^2 \rightarrow \mathbb{R}^2$$

A 3rd-year theorem,
you DON'T need to
know this yet

is a continuous function. The Borsuk-Ulam Theorem states that for any continuous function

$$g: S^2 \rightarrow \mathbb{R}^2,$$

there are at least two antipodal (opposite) points $x, y \in S^2$ such that $g(x) = g(y)$.

Hence for some pair of opposite points x, y on the Earth, $f(x) = f(y)$, proving the claim. \square

Mathematical Mythbusting

Everything published in mathematics is true. In particular, everything in a mathematics textbook is true.

Which one?

True

/

False

?

Fermat's Last Theorem

$x^n + y^n = z^n$ has no integer solutions for $n \geq 3$.

- Unproven for 300+ years
- Theorem proved by Andrew Wiles in the 90s.

Wiles first announced his proof on 23 June 1993 at a lecture in Cambridge entitled "Modular Forms, Elliptic Curves and Galois Representations".^[2]

However, in September 1993 the proof was found to contain an error. One year later on 19 September 1994, in what he would call "the most important moment of [his] working life", Wiles stumbled upon a revelation that allowed him to correct the proof to the satisfaction of the mathematical community.

The corrected proof was published in 1995.^[3]

From Wikipedia, "Wiles's Proof of Fermat's Last Theorem"

Types / Errors in Textbooks

NOT in the course, you don't need to know this.

But note there's an error every couple of pages

- Page 144: In the third paragraph of the proof of the rising sun lemma (Lemma 1.6.17), b should be b_n in the definition of A and in the next two occurrences (i.e. " t but not b " should be t but not b_n ", and " $t_* \in [t, b]$ " should be $t_* \in [t, b_n]$ ").
- Page 145, bottom: " $f'(x)$ exists" should be " $F'(x)$ exists". After Exercise 1.6.52, "ensure the almost everywhere existence" should be "ensure the absolute integrability of the derivative".
- Pages 149-152: In Section 1.7.1, "Caratheodory extension theorem" should be "Caratheodory lemma" throughout.
- Page 150, Exercise 1.7.2: "Lebesgue outer measurable" should be "the Lebesgue outer measure"
- Page 151: In the last two displays, and in the first display on the next page, $E_{N+1} \setminus \bigcup_{n=1}^N E_n$ may be simplified to E_{N+1} . In the second paragraph, "a disjoint sequence of" should be "a sequence of disjoint".
- Page 156: In Theorem 1.7.9, $-\infty < b < a < \infty$ should be $-\infty < a < b < \infty$. In the second paragraph of proof of this theorem, before ", adopting the obvious conventions", add "to be the required value of $\mu_F(I)$ given by (1.33) (e.g., $|[a, b]|_F = F_+(b) - F_-(a)$)".

From "An introduction to measure theory" by Terence Tao on Tao's website.

Takeaways:

- Mathematicians aren't perfect, sometimes we make mistakes

→ Never blindly trust mathematics. It is essential that you verify / check for yourself why something is true

→ If something doesn't make sense in a textbook, it could be a typo.

Sometimes even solutions have typos.

More Mythbusting

When mathematics is communicated, the only thing you need to worry about is whether or not it is correct. In particular, correct numerical solutions in exams automatically get 100%.

Which one: True / False

?

Question: Let $a \leq b \leq c$ be positive integers
(i.e. $1, 2, 3, 4, \dots$).

which satisfy

$$a + b + c = abc.$$

What is the maximum possible value of abc ? Explain your reasoning.

(3 min)

Solution: We first show that if a, b, c

(Note: this is hard to figure out in 5 min!)

satisfy the conditions of the question, then $ab \leq 3$.

(Note: this is hard to figure out in 5 min!)

$$\begin{aligned} \text{Since } a &\leq c, \\ b &\leq c, \\ c &\leq c, \end{aligned}$$

$$a + b + c \leq c + c + c = 3c$$

Since $abc = a + b + c$, this means

$abc \leq 3c$. Since c is positive, we can divide both sides by c and get $ab \leq 3$.

NOTE:

There is no expectation in this course that you can figure out a solution like this. It takes years of practice.

Solution:

Now we know $ab \leq 3$,
try and solve the problem.

(What's the maximum possible value of abc
if $abc = a + b + c$, $a \leq b \leq c$ are positive integers?)

NOTE: This part is
far less scary
than you may
think it is. You can all
do this. Really.

(2 min)

Expectations around Solutions

- Solutions should primarily convince the reader why something is true

→ Most questions will be graded as such

- In most 1000 / 2000 level math courses @ Cornell, solutions are graded based on
 - Completeness
 - Correctness

- Completeness

Did the student have a good go at solving the problem?

- Correctness

How convincing is the solution?

Let's grade some example solutions to the previous problem with these criteria.

Example Solution 1

We first show $ab \leq 3$.

Since $abc = a + b + c \leq 3c$, and $c \geq 0$,

$ab \leq 3$. Hence $(a,b) = (1,1), (1,2), (1,3)$.

Substituting these into $abc = a + b + c$,
the only possible value of c is 3.

Hence the max is $1 \cdot 2 \cdot 3 = 6$.

Completeness: /3

(Good go?)

Correctness: /3

(How convincing?)

Example Solution 2

It's possible to show that
 $abc = a + b + c$ implies $abc \leq 3$.

$$\therefore abc = 1, 2, 3.$$

$$\therefore (a, b) = (1, 1), (1, 2), (1, 3).$$

$$\text{If } (a, b) = (1, 1), c = c + 1$$

\therefore no solution

$$\text{If } (a, b) = (1, 2), 2c = c + 3$$

$$\therefore c = 3. \therefore abc = 6.$$

$$\text{If } (a, b) = (1, 3), 3c = c + 4$$

$$\therefore c = 2. \text{ but then } b > c.$$

\therefore Only solution is $(1, 2, 3)$

\therefore maximum is 6.

Completeness: /3

(Good go?)

Correctness: /3

(How convincing?)

Example Solution 3

6

Completeness: /3

(Good go?)

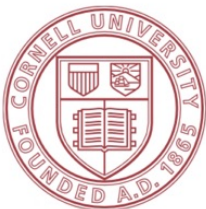
Correctness: /3

(How convincing?)

Takeaways

- Solutions in mathematics without an explanation usually have no value
- If asked to explain your reasoning, be sure to do so
- It's okay to write down partial ideas (e.g. "I tried to come up with a bound for a or b ") if you're unable to come up with a solution

- You will need to explain solutions ...



MATH 1110

Name: _____

Homework 1

NetID: _____

Friday, Sept 11

Worked with: _____

*If you have a printer or tablet computer, answer the questions in the spaces provided by printing this handout or annotating the pdf. If you do not have a printer or tablet, label and write each answer on a separate sheet of paper. In either case, your solutions should be hand-written (unless that is not possible). **Please include full explanations and write your answers using complete sentences.** When you have completed the assignment, upload all the pages to Gradescope by 11:59PM on the due date. Please make your grader's life easier by writing legibly, and make sure to "select pages" when uploading to Gradescope!!*

Final Myth

Mathematics is done by geniuses, usually alone. In particular, mathematicians know how to solve most problems quickly.

Which one
?

True / False

- Cutting-edge mathematics research is free: pretty much all mathematics papers are posted to arXiv.org.

The screenshot shows the top navigation bar of arXiv.org. On the left is the Cornell University logo and name. On the right, a text line reads: "We gratefully acknowledge support from the Simons Foundation and member institutions." Below this is a dark red header with the "arXiv.org" logo on the left and a search bar on the right. The search bar includes a search input field, a dropdown menu set to "All fields", and a "Search" button. Below the search bar are links for "Help" and "Advanced Search".

arXiv is a free distribution service and an open-access archive for 1,757,455 scholarly articles in the fields of physics, mathematics, computer science, quantitative biology, quantitative finance, statistics, electrical engineering and systems science, and economics. Materials on this site are not peer-reviewed by arXiv.

Subject search and browse:

Physics [Search] [Form Interface] [Catchup]

COVID-19 Quick Links

See COVID-19 SARS-CoV-2 preprints from

- [arXiv](#)
- [medRxiv and bioRxiv](#)

Important: e-prints posted on arXiv are not peer-reviewed by arXiv; they should not be relied upon without context to guide clinical practice or health-related behavior and should not be reported in news media as established information without consulting multiple experts in the field.

- It's actually hosted by Cornell!

- When searching mathematics papers on there, the majority of papers have several authors.

→ i.e. most mathematicians work on solving problems together.

- It's best to start early

→ find a classmate to work on problems with. (e.g. via Piazza)

You don't need to understand these words!

From arXiv ↘

1. arXiv:2007.09828 [pdf] math.GT

Over then Under Tangles

Authors: Dror Bar-Natan, Zsuzsanna Dancso, Roland van der Veen

Abstract: Brilliant wrong ideas should not be buried and forgotten. In underneath the layer of wrong. In this paper we explain how "over th and under the surface, also to some valid mathematics: a separation with an overview of other instances wh... ▾ More

Submitted 21 July, 2020; v1 submitted 19 July, 2020; originally announced July 20

2. arXiv:1910.00979 [pdf, ps, other] math.AG math.CO

Deletion-contraction triangles for Hausel-Proudfoot

Authors: Zsuzsanna Dancso, Michael McBreen, Vivek Shende

Abstract: To a graph, Hausel and Proudfoot associate two complex m of local systems on a Riemann surface, and moduli of Higgs bundles. which generalize local systems, and D carries the structure of a comp of these varieties count spanning s... ▾ More

Submitted 2 October, 2019; originally announced October 2019.

3. arXiv:1905.03067 [pdf, other] math.CO math.RA math.RT

Koszul Algebras and Flow Lattices

Authors: Zsuzsanna Dancso, Anthony M. Licata

Abstract: We provide a homological algebraic realization of the lattice edge-connected graph Γ with a spanning tree T , we associate a finite construction, planar dual graphs with dual spanning trees are associa category of finitely-generated... ▾ More

Submitted 28 May, 2019; v1 submitted 8 May, 2019; originally announced May 2

4. arXiv:1811.08558 [pdf, ps, other] math.QA math.GT

Ribbon 2-Knots, $1 + 1 = 2$, and Duflo's Theorem fo

Authors: Dror Bar-Natan, Zsuzsanna Dancso, Nancy Scherich

Abstract: We explain a direct topological proof for the multiplicativity

- Almost all of mathematics research is about solving problems no one has been able to solve yet

→ The idea that mathematicians know how to solve most problems quickly is false, simply because there are always more unsolved problems!

Math is a language

- Just as no one is born being able to speak a given language, no one is born being able to do math
- Reading a dictionary of another language will not make you fluent in that language... you need to practice!

- If you encounter words / symbols you don't know the meaning of, it's ultimately your responsibility to look them up.

$\ln(x)$

Derivative

Integral

differentiable

e

$\sin(x)$

function

continuous

π

Warning

For native speakers of English, translating another language into English is usually much easier than going in the opposite direction

We'll give you loads of practice to do this

→ Don't fall into the same trap with math!! Understanding a solution is easier than coming up with it yourself. You will be expected to come up with solutions. This only comes about with practice.

Math is hard

(Anyone who says otherwise is lying!)


- Are you expected to be able to solve every problem on your own without help?

Which one?

Yes / No

- Where can you get help? →

Math Support (not exhaustive)

- Other students
- Piazza
- Office hours
- Math support centre
- Online Q & A boards (e.g. Math Exchange)
- Student support groups 
- Tutorials / examples on Youtube

Some student STEM support groups

Association for Women in Mathematics (AWM)

Society of Women Engineers at Cornell (SWE)

Women In Computing at Cornell (WICC)

Association for Women in Science (ASWC)

Underrepresented Minorities in Computing (URMC)

ColorStack

The National Society of Black Engineers (NSBE)

Society of Hispanic Professional Engineers (SHPE)

American Indian Science & Engineering Society (AISES)

Cornell First Generation Students Union

Women's Resource Center

LGBT Resource Center

If those other options fail...

- Is it okay to search the internet for a solution to an assignment problem?

Yes / No

• Is it okay to use a solution to a math problem on the internet in an assignment?

Which one?
? Yes / No

• PROVIDED you:

- Cite the webpage you got the solution from
- Rewrite the solution in your own words
- Build on the solution to make a better solution

... it's okay for assignments.

It's never okay for exams.

Example

I actually got the question before from Math Exchange!

Maximum value of abc where $abc = a + b + c$ and a, b, c positive integers

Asked yesterday Active yesterday Viewed 26 times

- ▲ I came across an exercise in an algebra book which is the question in the title of this post.
- 1 ▼ We can assume WLOG that $a \leq b \leq c$. The book gave a hint: show that $ab \leq 3$.
- ★ I was able to figure out the answer using the hint: it implies $(a, b) = (1, 1), (1, 2), (1, 3)$, and then after substitution you get that $(a, b, c) = (1, 2, 3)$ is the only option with $b \leq c$.
- 🔄 I have no idea how to show $ab \leq 3$ though. How do I do this?

algebra-precalculus inequality

Features

- 📄 New
- 📄 Hot
- ⚙️ Guide

Related

?


1 Answer

Active Oldest Votes

- ▲ With $a \leq b \leq c$, we have
- 3

$$abc = a + b + c \leq c + c + c = 3c$$

share cite improve this answer follow

answered yesterday
 Hagen von Eitzen

How can this solution be improved?

- We all get stuck from time to time, solutions can point us in a good direction
- Provided all the previous criteria are met, you won't be penalized.
- A failure to cite a website, or a close collaboration with friend is an instance of plagiarism
- There is no need to cite course materials.

- The consequences for plagiarism can be severe.

- Failing a course
- A permanent record of cheating on your transcript.

- In the case of an exam, using non-approved internet resources is strictly prohibited.


Takeaways

- If you're stuck on an assignment problem, were unable to get help, and found a solution online:

- Cite the url

- Rewrite the solution

- Improve the solution

 Demonstrate you've thought about the problem.



From
the
Cornell
"cheating"
video
↙

- Mayor Myrick's thoughts on cheating
- You should know who he is!

Interactions with horrible people

If you discover someone is a terrible person, let Cornell know about it.



This guy would absolutely need to be reported & should be kicked out of Cornell.

From Rina Sawayama's excellent "STFU" Music Video available on YouTube.

Diversity and Inclusion



[Our Story](#)

[Belonging at Cornell](#)

[Our Community](#)

[Our Commitments](#)

Our Commitments

[Diversity Leadership at Cornell](#)

[DIWD](#)

[Inclusive Excellence Network](#)

[Home](#) / [Our Commitments](#) / Bias Reporting at Cornell

Bias Reporting at Cornell

[Report A Bias Incident](#)

Since 2000, Cornell University has had a program to track bias that is occurring on all campuses in an effort to be proactive in creating an inclusive climate for all. The [Department of Inclusion and Workforce](#)

Anyone
can use
this about
anyone
↙

It's not just bias

- It doesn't necessarily need to be intentional

↳ e.g. a professor saying "him or her" instead of the more inclusive "they".

A dropdown menu with a blue header bar and a list of categories. The categories are:

- Bias
- Criminal Activity
- Discrimination
- Harassment
- Domestic and Dating Violence
- Gender Based or Sexual Harassment
- Hazing
- Retaliation
- Sexual Assault or Misconduct
- Sexual Exploitation
- Stalking
- Violation of Interim Measure under Policy 6.4
- Workplace Climate Concerns
- Other/Not Sure Which Category to Select

Anonymity Options

Please remember that the timing and manner in which the University addresses this report will vary depending on the information provided and whether involved parties are available for further discussion.

- You may contact me
- To the extent possible, I would like to remain anonymous to involved individuals, but you may contact me
- Please do not contact me

Summary

- Why study math?
- Mathematics Mythbusting
 - Published math is always correct
 - Being correct is everything
 - Mathematics is done by solitary geniuses
- Math support groups
- ↳ Plagiarism & Cheating
 - Letting Cornell know about horrible people.
 - SAWAYAMA is an amazing album. Listen to it!

Next Time:

- Limits
- Continuity of functions

Upcoming Assessment

- HWO assignments from wk. 1 (Fri, 9/11)
- wk 2 Pre class activity (Thu, 9/10)
- Written HW (Fri, 9/11)
- online HW (Fri, 9/11)

Thanks for coming!

These slides are available
on Canvas:

Files → Thompson ⇒ DisSlides-Wk2.pdf