Practice Quiz and Solutions

Problems

1. Does $(x^3+y^3)^{1/3} = x+y$ for all x, y?

2. Does $p(x) = q(x)$, where $p(x) = (x-1)(x+2)(x-3)(x+4)$, and $q(x) = x^4+2x^3-13x^2-14x+26$?

3. Expand and simplify $f(x,y) = (x-y)(x^3+x^2y+xy^2+y^3)$.

4. Evaluate
$$\lim_{x\to 2} \frac{x^4-16}{x-2}$$
if it exists. If it does not, explain why.

5. Can k be chosen so that
$$f(x) = \begin{cases} (x^4-16)/(x-2) & x \neq 2 \\ k & x = 2 \end{cases}$$
is continuous? If so, what value? If not, why?

Solutions

1. Let $(x,y) = (1,1)$. Then $(x^3+y^3)^{1/3} = (1^3+1^3)^{1/3} = 2^{1/3} \neq 2 = 1+1$, so it is not the case that the $(x^3+y^3)^{1/3} = x+y$ holds for all x, y.

2. Note that $p(0) = (-1)(2)(-3)(4) = 24 \neq 26 = q(0)$, so $p \neq q$.

3. We have
$$\begin{aligned}(x-y)(x^3+x^2y+xy^2+y^3) &= x(x^3+x^2y+xy^2+y^3) - y(x^3+x^2y+xy^2+y^3) \\ &= x^4+x^3y+x^2y^2+xy^3-x^3y-x^2y^2-xy^3-y^4 \\ &= x^4-y^4.\end{aligned}$$
So in fact $f(x,y) = x^4-y^4$.

4. Note that $x^4-16 = x^4-2^4 = f(x,2)$, where $f(x,y)$ is the function in the previous question. We can therefore use the algebraic identity in the previous question to simplify this rational function:
$$\frac{x^4-16}{x-2} = \frac{(x-2)(x^3+2x^2+4x+8)}{x-2} = x^3+2x^2+4x+8.$$
Hence
$$\lim_{x\to 2}\frac{x^4-16}{x-2} = \lim_{x\to 2}(x^3+2x^2+4x+8) = (2)^3+2(2)^2+4(2)+8 = 32.$$

5. The function $(x^4-16)/(x-2)$ is a rational function, so it is continuous (cts) at all points where it is defined, i.e. when $x \neq 2$. Therefore $f(x)$ will be cts everywhere if and only if it is cts at $x = 2$.

   In order to be cts at $x = 2$, we need $f(2) = \lim_{x\to 2} f(x)$. We computed this limit in the previous question. Hence in order to be cts, we need $k = f(2) = \lim_{x\to 2}(x^4-16)/(x-2) = 32$. So we can choose $k = 32$ to make f cts.