MATH1920

Wk 1 Discussion 2/8/2021

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he/him

paper your: - Name - Preferred pronouns -Designated seat number

Please send me / write on

There's a good chance some of us will enter quarantine at some point...

- You can attend <u>any</u> online discussion in this case.
- You can attend <u>any</u> office hours more generally.

Discussion Outline

- Why study math?
- Mathematics mythbusting
 - Published math is always correct
 - Being correct is everything
 - Mathematics is done in isolation by geniuses
- Math support groups
- Plagiarism & Cheating
- Letting Cornell know about horrible people

Why Study Math?

- Some reasons: (not all!)
 - It's used throughout science
 - It's a CAS prerequisite (this is a valid reason!)
 - Problem solving is fun
 - You can use it to prove things about the world that are otherwise impossible to prove
 - Once something is proved it's true forever

Prove that at any given time, there are two opposite points on the surface of the Earth with the same temperature and pressure.

(1 min)

Consider the xy-plane.

Let the x-axis denote temperature.

Let the y-axis denote pressure.

Then the temperature and pressure at each point of the Earth is represented by some point in the xy-plane.

This is a function from the surface of the Earth to the xy-plane.

Since temperature and pressure vary continuously over the Earth, this function is continuous.

A 3rd-year theorem, you don't need to know it yet. The Borsuk-Ulam Theorem states that <u>any</u> continuous function from the sphere to the xy-plane has at least two opposite (antipodal) points which get sent to the same point in the plane.

Therefore there are at least two opposite points on the Earth which take the same pressure and temperature.

Everything published in mathematics is true. In particular, everything in mathematical textbooks is true.



Theorem: non-trivial

- $X^n + Y^n = Z^n$ has <u>no</u> integer solutions for $n \ge 3$.
 - Unproven for 300+ years
 - Proved by Andrew Wiles in the 90s.

Wiles first announced his proof on 23 June 1993 at a lecture in Cambridge entitled "Modular Forms, Elliptic Curves and Galois Representations".^[2] However, in September 1993 the proof was found to contain an error. One year later on 19 September 1994, in what he would call "the most important moment of [his] working life", Wiles stumbled upon a revelation that allowed him to correct the proof to the satisfaction of the mathematical community. The corrected proof was published in 1995.^[3]

From Wikipedia, "Wiles' Proof of Fermat's Last Theorem"

Typos / Errors in Textbooks

NOT in the course, you don't need to know this (obviously). But note there's an error every couple of pages.

- Page 144: In the third paragraph of the proof of the rising sun lemma (Lemma 1.6.17), b should be b_n in the definition of A and in the next two occurrences (i.e. "t but not b" should be t but not b_n ", and " $t_* \in [t,b)$ " should be $t_* \in [t,b_n)$ ".
- Page 145, bottom: "f'(x) exists" should be "F'(x) exists". After Exercise 1.6.52, "ensure the almost everywhere existence" should be "ensure the absolute integrability of the derivative".
- Pages 149-152: In Section 1.7.1, "Caratheodory extension theorem" should be "Caratheodory lemma" throughout.
- Page 150, Exercise 1.7.2: "Lebesgue outer measurable" should be " the Lebesgue outer measure"
- Page 151: In the last two displays, and in the first display on the next page, $E_{N+1} \setminus \bigcup_{n=1}^{N} E_n$ may be simplified to E_{N+1} . In the second paragraph, "a disjoint sequence of" should be "a sequence of disjoint".
- Page 156: In Theorem 1.7.9, $-\infty < b < a < \infty$ should be $-\infty < a < b < \infty$. In the second paragraph of proof of this theorem, before ", adopting the obvious conventions", add "to be the required value of $\mu_F(I)$ given by (1.33) (e.g., $|[a,b]|_F = F_+(b) F_-(a)$)".

From "An introduction to measure theory" by Terence Tao on Tao's website.

- Mathematicians aren't perfect (obviously), sometimes we make mistakes!
 - Never assume all mathematics is true! It is <u>essential</u> that you verify / check for yourself why something is true.
 - If something doesn't make sense in a textbook, it could be a typo. <u>Sometimes even solutions have typos.</u>

When mathematics is communicated, the only thing you need to worry about is whether or not it is correct.

In particular, correct numerical solutions in exams automatically get 100%



?

Which one

Let $A \le B \le C$ be positive integers (i.e. 1,2,3,...) which satisfy

A + B + C = ABC

What is the maximum possible value of ABC?

Explain your reasoning.



Solution

We first show that if A,B,C satisfy the conditions of the question, then $AB \le 3$.

Since



NOTE: There is no expectation in this course that you can figure out a solution like this in a few minutes. It takes years of practice.

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ABC = A + B + C \le C + C + C = 3C.
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Since C is positive, we divide both sides by C and get $AB \le 3$.

Solution (continued)

Now that we know $AB \leq 3$, try and solve the problem:

What's the maximum possible value of ABC if ABC = A + B + Cand $A \le B \le C$ are positive integers?



- Solutions should primarily <u>convince</u> the reader why something is true.
 - Most questions will be graded as such.
- In most 1000 / 2000 level math courses at Cornell, solutions are graded based on:
 - Completeness
 - Correctness

Expectations around solutions

• Completeness:

- Did the student have a good go at solving the problem?
- Correctness:
 - How convincing is the solution?

Let's grade some example solutions to the previous problem with these criteria.

We first show $AB \leq 3$.

Since ABC = $A + B + C \le 3C$ and $C \ge 0$, $AB \le 3$.

Hence (A,B) = (1,1), (1,2), (1,3).

Substituting these into ABC = A + B + C, the only possible value of C is 3. Hence the max is $1^{*}2^{*}3 = 6$.

Completeness: **3** / 3 (Good go?) Correctness₂/3

(How convincing?)

Example Solution 2

It's possible to show that ABC = A + B + C implies $AB \le 3$. So AB = 1,2,3. So (A,B) = (1,1),(1,2),(1,3). If (A,B) = (1,1), C = C + 2 \rightarrow no solution.

Completeness: **7** / 3 (Good go?)

If (A,B) = (1,2), 2C = C + 3So C = 3, so ABC = 6. If (A,B) = (1,3), 3C = C + 4So C = 2. But then B > C, which Is not allowed. \rightarrow Only solution is (1,2,3) \rightarrow Maximum is 6

Correctness **Z**/3

(How convincing?)

Example Solution 3

6

Completeness: () / 3 (Good go?)



(How convincing?)

- Solutions in mathematics without an explanation *are usually useless*.
- If asked to explain your reasoning, be sure to do so!
- It's okay to write down partial ideas (e.g. "I tried to come up with a bound for a or b") if you're unable to come up with a solution.

You will need to explain solutions in this course...

Math 1920

Draft: Vectors and Dot Products

HW#1

Please do the following text exercises from the class text. These will be graded for completion.



You answers to the following will be graded for correctness and quality of your solution.

Mathematics is done by geniuses, usually alone. In particular, mathematicians know how to solve most problems quickly.

Which one

True / False

?

• Cutting-edge mathematics research is free: pretty much all mathematical papers are posted to arxiv.org

We gratefully acknowledge supp the Simons Foundation and member inst				
				<u>Login</u>
arXiv.org		Search	All fields	Search
		Help Advanced Sea	rch	
arXiv is a free distribution service and an open-access archive for 1,833,965 scholarly articles in the fields of physics, mathematics, computer science, quantitative biology, quantitative finance, statistics, electrical engineering and systems science, and economics. Materials on this site are not peer-reviewed by arXiv.	COVID-19 Quick Links See COVID-19 SARS-CoV-2 preprints from • arXiv • medRxiv and bioRxiv			
Physics v Search Form Interface Catchup	should not be relied upon with	out context to guide	clinical praction	ce or health-

• It's actually hosted by Cornell!

Loads of authors...

From arXiv

- When searching mathematics papers on the arXiv, the majority of papers have several authors
 - I.e. most mathematicians work on solving problems together.
- It's best to start early!
 - Find a classmate(s) to work with on problems (e.g. via Ed Discussion)

Circuit algebras are wheeled props

Authors: Zsuzsanna Dancso, Iva Halacheva, Marcy Robertson

Abstract: Circuit algebras, introduced by Bar-Natan and the first author, are a gen Jones's planar algebras, in which one drops the planarity condition on "connectior provide a useful language for the study of virtual and welded tangles in low-dimer this note, we present the circuit algebra analogue of the well-known classification as pivot... ∇ More

Submitted 21 September, 2020; originally announced September 2020. Comments: 29 pages, many figures MSC Class: 57M25; 18D50

2. arXiv:2007.09828 [pdf, other] math.GT

Over then Under Tangles

Authors: Dror Bar-Natan, Zsuzsanna Dancso, Roland van der Veen

Abstract: Over-then-Under (OU) tangles are oriented tangles whose strands trave over crossings before any under crossings. In this paper we discuss the idea of gliby which any tangle diagram could be brought to OU form. Unfortunately, the alge However, by analyzing cases in which it does succeed we obtain a braid classificat also e... ∇ More

Submitted 4 February, 2021; v1 submitted 19 July, 2020; originally announced July 2020. Comments: 35 pages, lots of figures MSC Class: 57M25

3. arXiv:1910.00979 [pdf, ps, other] math.AG math.CO

Deletion-contraction triangles for Hausel-Proudfoot varieties Authors: Zsuzsanna Dancso, Michael McBreen, Vivek Shende

Abstract: To a graph, Hausel and Proudfoot associate two complex manifolds, B a

- Almost all of mathematics research is about solving problems no one has been able to solve yet.
 - The idea that mathematicians know how to solve most problems is false, simply because there are always more unsolved problems!

- Just as no one is born being able to speak a given language, no one is born being able to do math.
- Reading a dictionary of another language will not make you fluent in that language... you need to practice!

Math is a language (continued)

• If you encounter words / symbols you don't know the mean of in this course, ultimately it's your responsibility to look them up.

- For native speaker of English, translating another language into English is usually a lot easier than translating in the opposite direction.
 - Don't fall into the same trap with math:
 - Understanding a solution is easier than coming up with it yourself. You will be expected to come up with solutions.
 - This only comes about with practice.

What are some places you can get help with math?

Math Support (not exhaustive)

- Other students
- Ed Discussion
- Office hours
- Math support center
- Online Q&A boards (e.g. math exchange)
- Student support groups
- Tutorials / examples on Youtube / other platforms

Some student STEM support groups

- Association for Women in Mathematics (AWM)
- Society for Women Engineers at Cornell (SWE)
- Women in Computing at Cornell (WICC)
- Association for Women in Science (ASWC)
- Underrepresented Minorities in Computing (URMC)
- ColorStack
- The National Society of Black Engineers (NSBE)
- Society of Hispanic Professional Engineers (SHPE)
- American Indian Science & Engineering Society (AISES)
- Cornell First Generation Students Union
- Women's Resource Center / LGBT Resource Center

If you're finding it hard to get help...

Is it okay to search the internet for a solution to an assignment problem?



If you're finding it hard to get help...

Is it okay to use a solution to a math problem from the internet in an assignment?

Yes / No Hepends.

Terms and conditions:

PROVIDED you:

- Cite the webpage you get the solution from
- Rewrite the solution in your own words
- Build on the solution to make it better

... then it's okay for assignments. It's <u>**never**</u> okay for exams.

- We all get stuck from time to time, solutions can point us in a good direction. But remember:
- <u>A failure to cite</u> a website, or a close collaboration with another student, is an instance of plagiarism.
- There is no need to cite course materials (although you may find it useful.)

- The consequences of plagiarism can be severe:
 - Failing a course
 - A permanent record of cheating on your transcript
- In the case of an exam, using non-approved internet resources is strictly prohibited.

- If you're stuck on an assignment problem and were unable to get help with any of the previous resources, and have found a solution online:
 - Cite the url
 - Rewrite the solution in your own words
 - Improve it
 - Demonstrate that you've thought about the problem.



From the Cornell "cheating" video

- Mayor Myrick's thoughts on cheating.
- You should know who he is!

Interactions with horrible people

• If you discover someone at Cornell is a terrible person, let Cornell know about it.



This guy should > absolutely be reported & should be kicked out of Cornell.

 From Rina Sawayama's excellent "STFU!" music video. (Available on YouTube)

Diversity and Inclusion



Our Story	Belonging at Cornell	Our Community	Our Commitments		
Our Commitme	ents	r <u>Commitments</u> / Bias Reporting at Co	ornell		
Diversity Lead at Cornell	ership Bias F	Bias Reporting at Cornell			
DIWD		Report A Blas Incl	dent		
Inclusive Excel Network	llence Since 2000, Goccurring on	Cornell University has had a progran all campuses in an effort to be proad	n to track bias that is ctive in creating an		

inclusive climate for all. The Department of Inclusion and Workforce

Anyone can use this about anyone

It's not just bias

- It doesn't necessarily need to be intentional.
- E.g. a professor saying "him or her" instead of the more inclusive "they".

	•	
	Bias	
	Criminal Activity	
a	Discrimination	
4	Harassment	d in
a c	Domestic and Dating Violence	a in
t	Gender Based or Sexual Harassment	
	Hazing	
C	Retaliation	
	Sexual Assault or Misconduct	
	Sexual Exploitation	
	Stalking	
	Violation of Interim Measure under Policy 6.4	
	Workplace Climate Concerns	
	Other/Not Sure Which Category to Select	

Please remember that the timing and manner in which the University addresses this report will vary depending on the information provided and whether involved parties are available for further discussion.

○ You may contact me

○ To the extent possible, I would like to remain anonymous to involved individuals,

but you may contact me

○ Please do not contact me

- Why study math?
- Mathematics mythbusting (the following are false):
 - Published math is always correct
 - Being correct is everything
 - Mathematics is done is isolation by geniuses
- Math support groups
- Plagiarism & Cheating
- Letting Cornell know about horrible people
- SAWAYAMA is an amazing album.

Next Time

- Visualization!
- Vectors + Coordinates

- Complete the Welcome survey
- HW1

(Tuesday, 2/16)

Thanks for coming!

- These slides are linked on the Canvas course page under:
 Home
 - $\circ \rightarrow$ Zoom links for lectures, discussions, and office hours
 - $\circ \rightarrow$ Section Resources (under Ben Thompson)
 - $\circ \rightarrow \text{Week 1}$
 - $\circ \rightarrow$ Monday (slides)