Theorem 5.6.1 in the course notes is quite confusing on a first read. Let's work through some examples to get a better understanding of it.

## Question 1a

Let $T : R^2 \to R^2$ be the linear transformation which projects points in the plane to the y-axis. Let

$$b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

1. What is the matrix $A$ corresponding to $T$ (using the standard basis of $R^2$)?
2. Geometrically, describe the set of vectors $\{b - Av : v \in R^2\}$.
3. Geometrically, what is the smallest possible value of $||b - Av||$?
4. Give a geometric description of vectors $v$ which attain the minimum possible value above.

## Question 1b

Let

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

1. Is the equation Av = b consistent?
2. What is the normal equation in $z$ corresponding to $Av = b$?
3. Substitute $A, b$ into the normal equation in the previous step, and solve for $z$.
4. How do your $z$ values correspond to your geometric description in Question 1a?

Let's now apply Theorem 5.6.1 to calculate the distance between a point and a plane.

## Question 2

Consider the linear transformation $T : R^2 \to R^3$ given by matrix

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix},$$

and let

$$b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

1. What is the equation of the plane given by Im$A$?
2. Geometrically, what is the closest point to $b$ on the plane?
3. Is the equation $Ax = b$ consistent?
4. What is the normal equation in $z$ corresponding to this equation?
5. Solve the normal equation for $z$.
6. What is the value of $Az$? Is this value what you expected?

## Question 3

Let $(a, b), (c, d)$ be two points in $R^2$ with $a \neq c$, and let

$$A = \begin{bmatrix} 1 & a \\ 1 & c \end{bmatrix}, \quad q = \begin{bmatrix} b \\ d \end{bmatrix}.$$

1. What is the normal equation corresponding to $Ax = q$?
2. Solve the normal equation above.
3. What is the equation of the line through $(a, b)$ and $(c, d)$?
4. How is the line you found above related to your solutions of the normal equation?