Let's revise some of the course content so far to help prepare for the prelim tomorrow.

Question 1

What is an example of a 2×2 matrix which is:

1. not invertible and not diagonalizable?
2. not invertible but diagonalizable?
3. invertible but not diagonalizable?
4. invertible and diagonalizable?

Question 2

Let U be the set of all points in 3-dimensional Euclidean space that lie in the xy plane or the xz plane. Prove or disprove the following statement: U is a subspace of 3-dimensional Euclidean space.

Question 3

Consider the following vectors:

$$\alpha_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \alpha_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}, \quad \alpha_3 = \begin{bmatrix} 3 \\ 4 \\ 1 \\ 2 \end{bmatrix}, \quad \alpha_4 = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

1. Is the set $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ a basis for 4-dimensional Euclidean space?
2. Can the vector

   $$\beta = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

   be expressed an a linear combination of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$? If so, provide such an expression. If not, prove it is impossible to do so.

Question 4

Consider the plane in 3-dimensional Euclidean space given by the equation $x + 2y + 3z = 0$.

1. What is a 3×3 matrix whose null space is this plane?
2. What is a 3×3 matrix whose column space is this plane?

Question 5

Let A, B, C be 2×2 matrices, and let tr denote the trace of a matrix. Are the following true or false? Justify your answers.

1. $\text{tr}(ABC) = \text{tr}(CBA)$
2. $\text{tr}(ABC) = \text{tr}(BCA)$