Question 1

Let A, B be $n \times n$ matrices, and let $V = \{x \in R^n : Ax = Bx = 0\}$. Is V a subspace of R^n? Justify your answer.

Question 2

What is the 3×3 matrix which represents orthogonal projection in Euclidean space to the plane $x + y + z = 0$?

Question 3

Calculate the line of best fit between the following points in the Euclidean plane:

$$(-2, 4), \quad (0, 1) \quad (2, -1).$$

Question 4

Can the following sets of matrices be interpreted as vector spaces with the usual notion of matrix addition and scalar multiplication? If so, what is their dimension? Justify your answers.

1. The set of all 2×2 matrices whose diagonal entries sum to 0.
2. The set of all 2×2 matrices with entries even integers (including 0).
3. The set of matrices of the form
$$\begin{bmatrix} a & b \\ b & a \end{bmatrix}$$
where a and b are real numbers.

Question 5

Find a basis of solutions to the differential equation

$$y^{(3)} - y^{(1)} = 0$$

where y is a smooth function with domain and codomain the real numbers.