Question 1

Let a, b be two vectors in the Euclidean plane (R^2) which form angle θ. Prove that

$$a \cdot b = ||a|| \cdot ||b|| \cos\theta.$$

where $||a||$ and $||b||$ are the lengths of a, b respectively.

Question 2

Let T be a triangle in the Euclidean plane, and let a, b, c be vectors going from the origin to the corners of T.

1. Express the centroid of T in terms of a, b, c. (If you are unsure of the definition of the centroid of a triangle, check out the definition on Wikipedia.)

2. Let L be a line passing through a corner of T and its centroid. Let L_1 be the length of the line segment of L going from the corner to the centroid, and let L_2 be the length of the line segment of L going from the centroid to the side opposite the corner. Calculate the ratio $L_1 : L_2$.

Question 3

Given two intersecting planes in Euclidean space (R^3), their smaller angle of intersection is called their *dihedral angle*.

1. Calculate the dihedral axis of the xy-plane and the xz-plane.

2. Imagine two adjacent faces of a tetrahedron as planes. Calculate their dihedral angle.

3. Prove that the dihedral angle of the tetrahedron (calculated above) is not a rational multiple of π.

4. Prove that a cube cannot be cut up into small pieces and reassembled to form a tetrahedron. (Check out the Wikipedia page of the Dehn invariant.)