MATH 1106

Wk 1 Discussion 1/26/2024

Benjamin Thompson (he/they)

Cornell University

Slides Location

- These slides can be found at <u>bgthompson.com</u>:
 - $\circ \rightarrow$ Teaching [bar]
 - \circ → Modeling with Calculus for the Life Sciences (MATH 1106) Spring 2024 [link]
 - $\circ \rightarrow$ Wk1 Slides (F)

Discussion Outline

- What is the format of the discussion sections? Why come to them?
- What is it that mathematicians do?
- Admin
- Mathematics mythbusting
- Math support groups
- Plagiarism & Cheating
- D&I Bias reporting
- Worksheet

What is the format of these discussions?

- Worksheets, done collaboratively
- Quizzes

Today's a bit different though, the latter half will be course admin. (yay!)

Why bother coming to discussion sections?

Why bother coming to discussion sections?

- Collaborative It's significantly easier to collaboratively learn in person than online.
- Easier to learn math from people who are also learning it
- Diversity of perspectives everyone has different understandings of mathematics / ways of thinking
- Easy to ask questions / easy to get answers [sometimes :)]
- A consistent time / space to learn math
- Social!
- Many others...

- What is it that mathematicians do?
- Very difficult to come up with a precise answer...

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APPEARED IN BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY Volume 30, Number 2, April 1994, Pages 161-177

ON PROOF AND PROGRESS IN MATHEMATICS

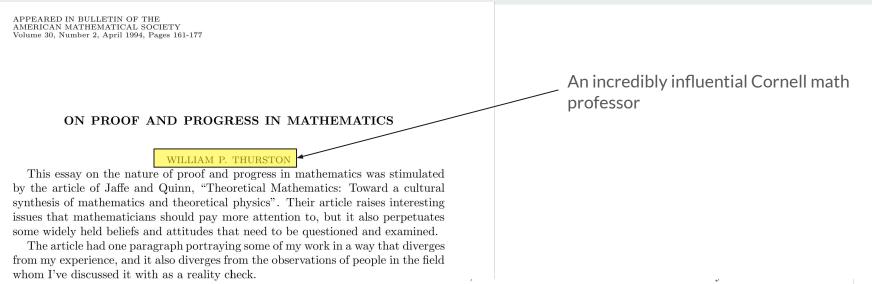
WILLIAM P. THURSTON

This essay on the nature of proof and progress in mathematics was stimulated by the article of Jaffe and Quinn, "Theoretical Mathematics: Toward a cultural synthesis of mathematics and theoretical physics". Their article raises interesting issues that mathematicians should pay more attention to, but it also perpetuates some widely held beliefs and attitudes that need to be questioned and examined.

The article had one paragraph portraying some of my work in a way that diverges from my experience, and it also diverges from the observations of people in the field whom I've discussed it with as a reality check.

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whom I've discussed it with as a reality check.



Could the difficulty in giving a good direct definition of mathematics be an essential one, indicating that mathematics has an essential recursive quality? Along these lines we might say that mathematics is the smallest subject satisfying the following:

- Mathematics includes the natural numbers and plane and solid geometry.
- Mathematics is that which mathematicians study.
- Mathematicians are those humans who advance human understanding of mathematics.

In other words, as mathematics advances, we incorporate it into our thinking. As our thinking becomes more sophisticated, we generate new mathematical concepts and new mathematical structures: the subject matter of mathematics changes to reflect how we think.

Calculation does not mean advancing human understanding

addition to knowledge that the theorem is true.

On a more everyday level, it is common for people first starting to grapple with computers to make large-scale computations of things they might have done on a smaller scale by hand. They might print out a table of the first 10,000 primes, only to find that their printout isn't something they really wanted after all. They discover by this kind of experience that what they really want is usually not some collection of "answers"—what they want is *understanding*.

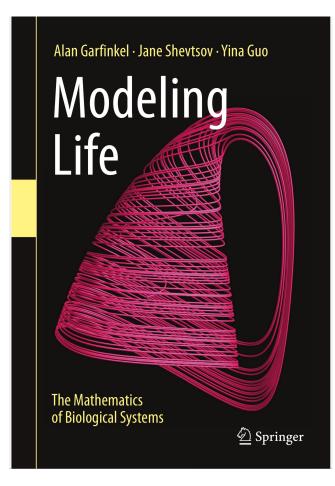
We want you to practice advancing human understanding!

LUNIV R	Math 1106	Name:
	Homework 2	NetID:
	Due $02/7$	Worked with:

Please complete these exercises, you may print this handout, annotate the PDF or write your answer on paper. When submitting your homework assign problems to your pages, see http://gradescope-static-assets.s3-us-west-2. amazonaws.com/help/submitting_hw_guide.pdf. It is important to be able to explain your reasoning to someone else in writing. Please include full explanations and write your answers using complete sentences (not just a bunch of mathematical symbols!). Points are specifically attributed to the clarity and accuracy of your explanations. Please upload all the pages of your homework to Gradescope.com by 11:00pm on the due date. Make your grader's life easier by writing neatly and legibly!!

- 2 Prelims (30%) on at: 7:30pm on: 03/05, and 04/11 [calendar]
- Final exam (25%) on at: TBD
- Written homework and Online Homework (25%)
- Quizzes (15%)
- Participation (5%)

What is the course textbook?



A .pdf can be downloaded from Cornell Library's webpage

What is the course textbook?

Modeling Life

The Mathematics of Biological Systems

"by Alan Garfinkel, Jane Shevtsov, Yina Guo"

Author, etc.:	Garfinkel, Alan, author Author info »	Availability
Format:	Book	
Language:	English.	✓Online
Edition:	1st ed. 2017.	<u>Access this title through Springer eBooks</u> (<u>Mathematics and Statistics</u>).
Published:	Cham : Springer International Publishing : Imprint: Springer, 2017.	> lerms of use
Subject:	Biomathematics. Mathematical models. Differential equations. Mathematical and Computational Biology. Mathematical Modeling and Industrial Mathematics. Differential Equations.	Access this title through Springer Nature eBooks & Books Series (LALC). > Terms of use Restrictions "License restrictions may limit access."
Summary:	From predator-prey populations in an ecosystem, to hormone regulation within the body, the natural world abounds in dynamical systems that affect us profoundly. This book develops the mathematical tools essential for students in the life sciences to describe these interacting systems and to	

Homework deadlines

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Calenda Common Participation Calenda Common Participation Common	rd ar	Spring 2024 Home Announcements Syllabus Modules Gradescope Grades Zoom People Assignments Collaborations Rubrics BigBlueButton Discussions Files Outcomes Pages Quizzes Settings		Wiew All Pages HOMEWORK Homework Schedule 1 1 2 Feb7 3 Feb14 4 Feb28 6 Mar13 7 Mar20 8 Mar27 9 Apr17 10 Apr24 11 May1 12 May7 (Shorter HW) Part of you homework will be completed of login to WeBWork, try again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the the login to WeBWork again in 24hours, of the	on We	BWorK where you will get automatic feed does not work, contact math1106-staff@c	back. You ma			Publisi	ned ۹		
				Here is a <u>PDF</u> \checkmark of homework, solutions of Previous	WIII DE	posted to this page after the due date.						Next •	•

Homework deadlines

Homework:

- There will be 12 homework sets posted on Canvas at least one week before their due date and are to be completed on WeBWorK and turned in on Gradescope by 11:00pm on Wednesday. The work you hand in should represent your own efforts and understanding. Don't risk an academic integrity violation by handing in copied solutions.
- Also note that any work you submit is a document that another person has to read, and the teaching team is not obliged to read unreasonably messy or disorganized submissions.
- You are all permitted to miss two homework sets, to account for unexpected situation, do not email the instructor or TAs about missed/incomplete homework. Please plan carefully and save your "missed" for illness or emergencies.
- In addition to missed homework, you will get to submit homework up to 48hrs late with **penalties**.

Everything published in mathematics is true. In particular, everything in mathematical textbooks is true.

Which one True / False

?

Theorem:

 $X^n + Y^n = Z^n$ has <u>no</u> non-zero integer solutions for $n \ge 3$.

- Unproven for 300+ years
- Proved by Andrew Wiles in the 90s.

Wiles first announced his proof on 23 June 1993 at a lecture in Cambridge entitled "Modular Forms, Elliptic Curves and Galois Representations".^[2] However, in September 1993 the proof was found to contain an error. One year later on 19 September 1994, in what he would call "the most important moment of [his] working life", Wiles stumbled upon a revelation that allowed him to correct the proof to the satisfaction of the mathematical community. The corrected proof was published in 1995.^[3]

From Wikipedia, "Wiles' Proof of Fermat's Last Theorem"

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Typos / Errors in Textbooks

NOT in the course, you don't need to know this (obviously). But note there's an error every couple of pages.

- Page 144: In the third paragraph of the proof of the rising sun lemma (Lemma 1.6.17), b should be b_n in the definition of A and in the next two occurrences (i.e. "t but not b" should be t but not b_n ", and " $t_* \in [t,b)$ " should be $t_* \in [t,b_n)$ ".
- Page 145, bottom: "f'(x) exists" should be "F'(x) exists". After Exercise 1.6.52, "ensure the almost everywhere existence" should be "ensure the absolute integrability of the derivative".
- Pages 149-152: In Section 1.7.1, "Caratheodory extension theorem" should be "Caratheodory lemma" throughout.
- Page 150, Exercise 1.7.2: "Lebesgue outer measurable" should be " the Lebesgue outer measure"
- Page 151: In the last two displays, and in the first display on the next page, $E_{N+1} \setminus \bigcup_{n=1}^{N} E_n$ may be simplified to E_{N+1} . In the second paragraph, "a disjoint sequence of" should be "a sequence of disjoint".
- Page 156: In Theorem 1.7.9, $-\infty < b < a < \infty$ should be $-\infty < a < b < \infty$. In the second paragraph of proof of this theorem, before ", adopting the obvious conventions", add "to be the required value of $\mu_F(I)$ given by (1.33) (e.g., $|[a,b]|_F = F_+(b) F_-(a)$)".

From "An introduction to measure theory" by Terence Tao on Tao's website.

- Mathematicians aren't perfect (obviously), sometimes we make mistakes!
 - Never assume all mathematics is true! It is <u>essential</u> that you verify / check for yourself why something is true.
 - If something doesn't make sense in a textbook, it could be a typo. <u>Sometimes even solutions have typos.</u>

When mathematics is communicated, the only thing you need to worry about is whether or not it is correct.

In particular, correct numerical solutions in exams automatically get 100%

Which one

True / False

?

Let $A \le B \le C$ be positive integers (i.e. 1,2,3,...) which satisfy

A + B + C = ABC

What is the maximum possible value of ABC?

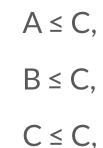
Explain your reasoning.



Solution

We first show that if A,B,C satisfy the conditions of the question, then $AB \le 3$.

Since



NOTE: There is no expectation in this course that you can figure out a solution like this in a few minutes. It takes years of practice.

```
ABC = A + B + C \le C + C + C = 3C.
```

Since C is positive, we divide both sides by C and get $AB \le 3$.

Solution (continued)

Now that we know $AB \leq 3$, try and solve the problem:

What's the maximum possible value of ABC if ABC = A + B + Cand $A \le B \le C$ are positive integers?



- Solutions should primarily <u>convince</u> the reader why something is true.
 - Most questions will be graded as such.
- In most 1000 / 2000 level math courses at Cornell, solutions are graded based on:
 - Completeness
 - Correctness

Expectations around solutions

• Completeness:

- Did the student have a good go at solving the problem?
- Correctness:
 - How convincing is the solution?

Let's grade some example solutions to the previous problem with these criteria.

We first show $AB \leq 3$.

Since ABC = $A + B + C \le 3C$ and $C \ge 0$, $AB \le 3$.

Hence (A,B) = (1,1), (1,2), (1,3).

Substituting these into ABC = A + B + C, the only possible value of C is 3. Hence the max is $1^{*}2^{*}3 = 6$.

Completeness: / 3

(Good go?)

Correctness / 3

(How convincing?)

Example Solution 2

It's possible to show that ABC = A+ B + C implies AB \leq 3. So AB = 1,2,3.So(A,B) = (1,1), (1,2), (1,3).If (A.B) = (1.1), C = C + 2 \rightarrow no solution.

Completeness: / 3 (Good go?)

If (A,B) = (1,2), 2C = C + 3So C = 3, so ABC = 6. If (A,B) = (1,3), 3C = C + 4So C = 2. But then B > C, which Is not allowed. \rightarrow Only solution is (1,2,3) \rightarrow Maximum is 6 Correctness / 3

(How convincing?)

Example Solution 3

6

Completeness: / 3 (Good go?) Correctness / 3 (How convincing?)

- Solutions in mathematics without an explanation *are usually difficult to understand*.
- If asked to explain your reasoning, be sure to do so!
- It's okay to write down partial ideas (e.g. "I tried to come up with a bound for a or b") if you're unable to come up with a solution.

Mathematics is done by geniuses, usually alone. In particular, mathematicians know how to solve most problems quickly.

Which one

?

True / False

• Cutting-edge mathematics research is free: pretty much all mathematical papers are posted to arxiv.org

We gratefully acknowledge suppo the Simons Foundation and member instit					
				Lo	<u>ogin</u>
arXiv.org		Search	All fields	🗸 Sear	rch
5		Help Advanced Sea	rch		
arXiv is a free distribution service and an open-access archive for 1,833,965 scholarly articles in the fields of physics, mathematics, computer science, quantitative biology, quantitative finance, statistics, electrical engineering and systems science, and economics. Materials on this site are not peer-reviewed by arXiv.	COVID-19 Quick Links See COVID-19 SARS-CoV-2 • arXiv • medRxiv and bioRxiv			Viu łbou	
Physics v Search Form Interface Catchup	Important: e-prints posted on should not be relied upon with related behavior and should n	out context to guide	clinical practic	ce or health-	-

• It's actually hosted by Cornell!

Loads of authors...

From arXiv

• When searching mathematics papers on the arXiv, the majority of papers have several authors

- I.e. most mathematicians work on solving problems together.
- It's best to start early!
 - Find a classmate(s) to work with on problems (e.g. via Ed Discussion)

Circuit algebras are wheeled props

Authors: Zsuzsanna Dancso, Iva Halacheva, Marcy Robertson

Abstract: Circuit algebras, introduced by Bar-Natan and the first author, are a gen Jones's planar algebras, in which one drops the planarity condition on "connectior provide a useful language for the study of virtual and welded tangles in low-dimer this note, we present the circuit algebra analogue of the well-known classification as pivot... ∇ More

Submitted 21 September, 2020; originally announced September 2020. Comments: 29 pages, many figures MSC Class: 57M25; 18D50

2. arXiv:2007.09828 [pdf, other] math.GT

Over then Under Tangles

Authors: Dror Bar-Natan, Zsuzsanna Dancso, Roland van der Veen

Abstract: Over-then-Under (OU) tangles are oriented tangles whose strands trave over crossings before any under crossings. In this paper we discuss the idea of gliby which any tangle diagram could be brought to OU form. Unfortunately, the alge However, by analyzing cases in which it does succeed we obtain a braid classificat also e... ∇ More

Submitted 4 February, 2021; v1 submitted 19 July, 2020; originally announced July 2020. Comments: 35 pages, lots of figures MSC Class: 57M25

3. arXiv:1910.00979 [pdf, ps, other] math.AG math.CO

Deletion-contraction triangles for Hausel-Proudfoot varieties Authors: Zsuzsanna Dancso, Michael McBreen, Vivek Shende

Abstract: To a graph, Hausel and Proudfoot associate two complex manifolds, B a

- Almost all of mathematics research is about solving problems no one has been able to solve yet.
 - The idea that mathematicians know how to solve most problems is false, simply because there are always more unsolved problems!

- Just as no one is born being able to speak a given language, no one is born being able to do math.
- Reading a dictionary of another language will not make you fluent in that language... you need to practice!

Math is a language (continued)

• If you encounter words / symbols you don't know the mean of in this course, ultimately it's your responsibility to look them up.

- For native speaker of English, translating another language into English is usually a lot easier than translating in the opposite direction.
 - Don't fall into the same trap with math:
 - Understanding a solution is easier than coming up with it yourself. You will be expected to come up with solutions.
 - This only comes about with practice.

Math is hard; anyone who says math is easy is a liar. What are some places you can get help with math?

Getting support with math is not as hard as getting in here...



Math Support (not exhaustive)

- Other students
- Ed Discussion
- Office hours
- Math support center
- Online Q&A boards (e.g. math exchange)
- Student support groups
- Tutorials / examples on Youtube / other platforms

Some student STEM support groups

- Association for Women in Mathematics Cornell (AWM)
- <u>Society for Women Engineers at Cornell (SWE)</u>
- Women in Computing at Cornell (WICC)
- <u>Underrepresented Minorities in Computing Cornell</u>
 <u>(URMC)</u>
- <u>ColorStack</u>
- National Society of Black Engineers Cornell (NSBE)
- Society of Hispanic Professional Engineers Cornell (SHPE)
- American Indian Science & Engineering Society (AISES)
- <u>First Generation Students Union Cornell</u>
- Haven // gStem (LGBTQI+ in STEM)
- LGBT Resource Center

- The consequences of plagiarism can be severe:
 - Failing a course
 - A permanent record of cheating on your transcript
- In the case of an exam, using non-approved internet resources is strictly prohibited.

- If you're stuck on an assignment problem, seek help! There are LOADS of ways to get help in the course.
- Highly, highly recommended way: <u>the support course</u>!

Extra Resources

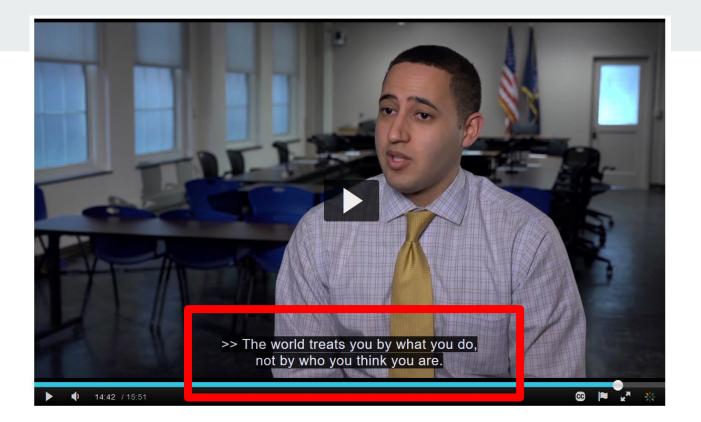
Support Course by LSC

Everyone is welcome to come to the support course for MATH1106, run by Dr. Fraser. The support course meets once a week (either M 1:25-2:40 MLT253 or W 2:55-4:10 MLT251), and is a time to review what you have done in class that

week, answer any questions you may have, and do extra practice problems. You do not need to be enrolled in the course to attend - just drop by whenever you'd like some extra help! You are also welcome to access extra problems and solution sets on the support course Canvas page, which you can find here.

Math Support Center

http://uann.pair.com/ref/msc/when/ ⊟→



From the Cornell "cheating" video

• Former Mayor Myrick's thoughts on cheating.

Not all faculty think a lot about D&I...

Divers	ity and Inc	clusion		
Our Story	Belonging at Cornell	Our Community	Our Commitments	
Our Commitm		r Commitments / Bias Reporting at	Cornell	Anyone can use this about anyone
Diversity Lea at Cornell	adership	Reporting at C		
DIWD		Report A Bias Inc	lident	
Inclusive Exc Network	occurring on	Cornell University has had a progr all campuses in an effort to be pro nate for all. The <u>Department of Inc</u>	pactive in creating an	

It's not just bias

- It doesn't necessarily need to be intentional.
- E.g. a professor saying "him or her" instead of the more inclusive "they".

	•	
	Bias	
а	Criminal Activity	
	Discrimination	
4	Harassment	d in
d	Domestic and Dating Violence	u III
f	Gender Based or Sexual Harassment	
	Hazing	
C	Retaliation	
	Sexual Assault or Misconduct	
	Sexual Exploitation	
	Stalking	
	Violation of Interim Measure under Policy 6.4	
	Workplace Climate Concerns	
	Other/Not Sure Which Category to Select	

Please remember that the timing and manner in which the University addresses this report will vary depending on the information provided and whether involved parties are available for further discussion.

○ You may contact me

○ To the extent possible, I would like to remain anonymous to involved individuals,

but you may contact me

○ Please do not contact me

- What is the format of the discussion sections? Why come to them?
- What is linear algebra?
- Admin
- Mathematics mythbusting
- Math support groups
- Plagiarism & Cheating
- D&I Bias reporting